Mathematics T (954)

OVERALL PERFORMANCE

The number of candidates for this subject was 8592. The percentage of candidates who obtained a full pass was 67.09%, an increase of 1.14% compared with the previous year.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	8.01	7.18	9.61	11.22	11.08	10.74	9.25	3.09	3.45	3.34	23.03

RESPONSES OF CANDIDATES

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General comments

Generally, the majority of answers were well written. However, quite a number of answers were not well thought off and the logical sequence was not there.

Comments on individual questions

Question 1

Almost all candidates were familiar with the property $\log ab = \log a + \log b$, but most of them made mistakes as they thought that $\log x^2 = (\log x)^2$ and $\log x + \log y = 5 \Leftrightarrow x + y = 5$.

Answers: $x = 27$, $y = 9$

Question 2

Most candidates were able to find $\dfrac{du}{dx}$ and used the chain rule. However, only a few were able to simplify $\sqrt{u^2 - 1}$ or $\dfrac{du}{dx}$ in terms of x correctly, and express $u^2 - 1$ as a perfect square. Since the solution required clear proof, the steps involved should have been detailed, but many candidates just concluded with the answer.

Question 3

Some candidates were aware of the condition for a convergent series which was $|r| < 1$, but they failed to realise that $e^{-x} > 0$ for all real values of x and thus, they did not answer the first part correctly. As for the second part, more than half of candidates did not provide the exact value as required by the question.

Answers: $\{x: x > 0\}$, $x = \ln \dfrac{4}{3}$

Question 4

Most candidates were able to find $f'(x)$. However, the majority of candidates failed to observe that the next part requires them to use answer from the first part. These candidates did not use the anti-derivative concept to obtain the answer but used integration by parts instead.

Answers: $f'(x) = 1 + \ln x$, $2e \ln e$

Question 5

There were only a few candidates who were able to use the definition for $A - B$ to establish the proof for the first part.

Answer: $k = \frac{2}{3}$

Question 6

Candidates could find the domain and range but could not gave the answer in the correct set form, or vice versa. Majority of the candidates were also very weak in giving explanation and reasoning.

Answers: $D_f = [-3, -1) \cup (-1, 2]$, $R_f = (-1, 2)$

Question 7

Most candidates were able to answer the first part, but failed to solve the inequality $p(x) \leqslant 0$ in the second part. Most candidates just showed that $x^2 - 4x + 5 = 0$ had complex roots without realising that it was always positive. Some candidates just ignored the quadratic factor saying that it had complex roots.

Answers: $a = 9$, $b = -5$, $\{x : -1 \leqslant x \leqslant \frac{1}{2}\}$

Question 8

Most candidates were able to find the constants A, B, C and D. However, there were some who made careless mistakes in solving the equations. There were also instances where candidates did not use the "≈" sign correctly and at the appropriate places.

Answers: $A = 1$, $B = -3$, $C = 4$, $D = 0$

Question 9

Most candidates were able to sketch the graph of $y = e^{-x}$, but they were not able to sketch the graph of $y = \frac{4}{2 - x}$. In many cases, candidates only drew part of the rational function which lay in the first and second quadrant. Most candidates were able to use the Newton-Raphson method correctly to estimate the root of the equation.

Question 10

Candidates did not performed well in this question. Some candidates were not aware of the properties of the circle. Very few candidates were able to answer the last part of the question since they did not realise that the two circles actually touched each other and hence, the intersection point was the midpoint of the two centres.

Answers: (*a*) $(x + 2)^2 + (y - 3)^2 = 25$; (*b*) $(-2, 3)$, 5; (*c*) (i) 10, (ii) $(1, 7)$

Question 11

Quite a number of candidates were able to find the composite function and stationary points correctly and to sketch the graph. However, some candidates failed to give the overall graph and presented only partial sketch or incomplete graph. For example, the sketch terminated in the second quadrant and did not proceed to the third quadrant. As for the second part, many candidates failed to sketch the graph $y = \frac{1}{h(x)}$ correctly. As a result, they were not able to give the right answers for part (*c*). Some candidates did not give the answers in the set form.

Answers: (*a*) $(x-1)^3 - 3(x-1) + 2$, $(0, 4)$, $(2, 0)$; (*c*) (i) $\left\{k : k < \frac{1}{4}, k \neq 0\right\}$, (ii) $\left\{k : k = \frac{1}{4}\right\}$, (iii) $\left\{k : k > \frac{1}{4}\right\}$

Question 12

All candidates knew that $\mathbf{P}^{-1} = \frac{1}{|\mathbf{P}|}$ Adj $\mathbf{P}$. However, quite a number of candidates confused with transpose of a matrix, adjoint and cofactors.

Some candidates were weak in transforming the information given into a system of linear equations. The candidates straight away wrote the matrix equation which they inadvertently missed showing the transformation of worded problems into mathematical equations clearly.

Answers: (*a*) $|\mathbf{P}| = 14$, Adj $\mathbf{P} = \begin{pmatrix} 2 & 1 & 5 \\ 8 & -3 & -1 \\ -4 & 5 & -3 \end{pmatrix}$, $\begin{pmatrix} \frac{1}{7} & \frac{1}{14} & \frac{5}{14} \\ \frac{4}{7} & -\frac{3}{14} & -\frac{1}{14} \\ -\frac{2}{7} & \frac{5}{14} & -\frac{3}{14} \end{pmatrix}$

(*b*) (ii) $x = 5$ minutes, $y = 8$ minutes, $z = 10$ minutes

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General comments

In general, the quality of answers given was not satisfactory. Candidates showed a wide range of mathematical abilities. Good candidates gave well-planned answers and systematic steps in their presentations. On the other hand, weaker candidates showed a lack of aptitude and understanding of the requirements of the questions.

Comments on individual questions

Question 1

This question was well answered by most candidates. However, in the integration by parts, many candidates made a wrong choice of u and $\frac{du}{dv}$, thus making the integration more difficult to solve. Another common mistake was $\int \cos y \, dx = -\sin y$.

Answer: $\sin y = x^2 \ln x - \frac{1}{2}x^2 + \mathrm{c}$

Question 2

Most candidates were able to answer the first part of the question, but failed to solve the second part of the question. They were confused about the maximum and minimum values and failed to make use of 13 $\sin(\theta + \alpha) + 15$. Another common mistake was $\frac{1}{2} \leqslant \frac{1}{5 \sin \theta + 12 \cos \theta + 14} \leqslant \frac{1}{28}$.

Answer: Maximum value $= \frac{1}{2}$ and minimum value $= \frac{1}{28}$

Question 3

Very few candidates were able to solve part (*a*) and part (*b*). Mistakes were made when candidates assumed PB = OB = OA.

Question 4

Generally, this question was well answered. However, a number of candidates were able to prove that $vx\frac{dv}{dx} = 1$, but they did not use it to solve the given differential equation. Instead, they made a futile attempt to solve it using the original differential equation.

Answer: $y^2 = 2x^2 (\ln x + 2)$

Question 5

This question was poorly answered. Many candidates were able to sketch the diagram, but they could not proceed to argue the proof and wrote many irrelevant properties needed for the proof. There were a number of candidates who confused congruent triangle with similar triangle. Not many candidates were able to answer part (*d*) due to their failure to apply the properties of similar triangles to help them.

Question 6

This was a difficult question for candidates. They were able to answer parts (*a*) and (*b*), but they failed to pursue the subsequent parts. They also failed to apply the ratio theorem correctly. Most candidates expressed $\overrightarrow{OL}$ in terms of $\overrightarrow{OQ}$, and only a few were able to find $PM : MQ$.

Answers: (*a*) $\overrightarrow{ON} = \frac{2}{5}\text{q} + \frac{3}{5}\text{r}$; (*c*) $QO : OL = 4:1$; (*d*) $PM : MQ = 2:5$

Question 7

This was an easy question for most candidates who had understood the underlying concepts. Mistakes were made by the candidates who assumed that R and U' were independent, thus wrote them as $P(R \cap U') = P(R)P(U')$. Among other mistakes were the interpretation of the question to which many were trying to find $P(R/U')$ or $P(U/R)$ or $P(U'/R)$ instead. Some even misunderstood $P(R \cap U')$ for $P(U'/R)$.

Answer: 0.6

Question 8

Many candidates failed to understand the concept of absolute value in the context of the question, and hence were not able to solve the question correctly. They knew the need to standardise the variable and many obtained just 1 mark in this process. If they had understood the modulus sign, they probably could have scored more marks in this question.

Answer: The least integer k is 11

Question 9

Many candidates were unable to obtain the probability distribution table from the cumulative distribution function, while many treated the variables as continuous. Nevertheless, most of them used the correct formulae to calculate the mean and variance.

Answer: (*b*) $\mathrm{E}(X) = 2$, $\mathrm{Var}(X) = 1.8$

Question 10

In general, the performance of candidates on this question was moderate. Though, many knew that the integral of the density function is the cumulative distribution, many were not sure of the limit of the integral that was required. The integral required integration by parts and many made the mistake of the wrong choice of the variables of substitution. Many candidates did not relate $\mathrm{P}(X > 9) = 1 - \mathrm{P}(X < 9)$, thus using $\mathrm{F}(9)$ as $\mathrm{P}(X > 9)$.

Answer: (*b*) 0.199

Question 11

This question was generally well answered. Many candidates knew the binomial experiment and its approximate distribution. Those who failed to answer were those who could not interpret "at least one" and those who had assumed non-binomial distribution.

Answers: (*a*) Maximum number of eggs is 28; (*b*) 0.143

Question 12

This represented quite an easy plot question, but required analytical reasoning to answer some of the questions posed. Nevertheless, there were candidates who used wrong upper values and many used mid-points to plot ogive. Weakness in reading values from the graph was quite apparent in some candidates and some were able to 'read' up to 3 decimal places. In parts (*c*) and (*d*), candidates failed to justify their comments, thus losing marks.

Answers: (*b*) median = 6.10, semi-interquartile range = 0.5; (*d*) (i) 18.18%; (ii) Median = 6.20