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LAPORAN PEPERIKSAAN STPM

Mathematics (T) (954)

Mathematics (T) 954/1

OVERALL PERFORMANCE

The number of candidates for this subject was 2 733. The percentage of candidates who obtained a full pass was 38.38%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	3.63	2.13	3.29	4.75	6.47	8.31	9.80	7.91	8.42	7.60	37.69

RESPONSES OF CANDIDATES

PAPER 954/1

General comments

All the candidates answered in English but their presentation varied from one candidate to the other with spelling and grammar mistakes. In general, all candidates' answers were satisfactory but their performance differs from one candidate to the others which reflect the wide range of the mathematical ability. Even though quite a number of candidates arranged the answers in proper order and managed to write well-presented and neat answers but a small number of candidates still produced messy and unorganised works which showed unplanned working. Overall, the presentation of the answers were not as good as in previous years.

There were a mixed quality of answers; good, moderate and weak performances by candidates in terms of their knowledge, mathematical ability to apply formulae, capacity to reason and make deduction. There were candidates with good knowledge and ability to apply formulae as well as perform reasoning and deduction, however, in this year's performance, there were a higher percentage of candidates which lack basic knowledge, poor mathematical ability and unable to reason and deduce.

Good candidates were able to plan and gave well-organised answers but the numbers were very few. They showed all the essential workings, their answers were systematic, logic and well presented with the right concepts and laws. Their handwriting were neat, clear and easily understood. The hyperbola sketched by them was accurate and correctly labeled. They were able to give appropriate descriptions and explanations as requested in the questions. They showed good understanding in using "hence". Excellent candidates could answer all question with complete and well-presented answers.

Moderate candidates were able to present their answers well for the questions or the parts they were familiar with. Most of them managed to get the first part or some parts correct in their workings. Quite a number of candidates chose not to answer particular questions, whereas some tried to answer and wrote a lot, but all were irrelevant. They could not answer those questions that require further knowledge and applications of the topics such as questions 2, 3, 5, 6 and 8.

Weak candidates have poor foundations in basic mathematical concepts, lacked the understanding of what were required in the questions and thus lacked the aptitude to apply the appropriate concept to solve the problem. It was apparent from their answers, they performed badly on presenting solutions which entail reasoning, deduction and sketching of geometrical shape. Some did not answer according to the requirement of the questions.

Their workings are not well-planned. Referring to the candidates' solutions, majority of them failed to show essential steps to convince as to how the answers came about. Answers given by them were disorganised and not properly presented. Almost all of the weak candidates failed to solve the questions involving the instruction of "hence". This can be seen in their attempts in answering questions 2 and 7.

On another note, in terms of presentation of answers, there are still candidates who wrote their answers in two columns presenting their working, split a page to two, squeezing two questions to one side making a page with solutions of 3 to 4 questions which make it hard for the examiners to write marks at the appropriate place. Some candidates still produced small and unorganised set of answers, consequently their scripts were totally packed and difficult to mark.

Comments on individual questions

Question 1

Most of the candidates who attempted this question were able to use the Remainder and Factor Theorem to set up simultaneous equations, ultimately determining the values of m and n. The majority successfully performed long division to find the remainder, though some candidates opted to compare coefficients to derive it. The equation $x^4 - 11x^2 - 18x - 8 = (x^2 - 1)(x^2 + bx + c) + px + q$ or $\mathrm{p}(x) = (x^2 - 1)\mathrm{f}(x) + q(x)$, where $q(x) = ax + b$, allowed for easy remainder calculation by substituting $x = -1$ and $x = 1$, to form two linear equations. A few candidates, however, failed to recognise the use of the Remainder and Factor Theorem, which prevented them from completing the task of finding the values of m and n. Many candidates who successfully used long division and found the remainder still made errors when determining $q(x)$, leading to incorrect conclusions.

Answers: $m = 1$, $n = -11$; $q(x) = -18x - 18$

Question 2

A good number of candidates could answer the first part superbly. Since this question is a familiar one, candidates took the chances to score marks as much as possible. On the other hand, the second part was not contributing marks to the average and weak candidates since only a few of them successfully related the term of sequence attained with the second part, which is to find the sum of a specified series. Most candidates excellently identified the given sequence as an arithmetic progression to obtain the n^{th} term correctly by using the general term of arithmetic progression. Candidates' carelessness when not placing brackets appropriately when substituting $r + 1$ in u_r to get the term u_{r+1}. Quite a number of the candidates were unable to relate the nth term of the given sequence attained with the last part which is to find the sum of a specified series. Candidates failed to express the sum of the series using summation notation as $\sum_{r-1}^{100} r(6r - 2)$. Some candidates did not attempt this part of evaluating the given series mainly due to not obtaining the general term of the given series as $r(6r - 2)$. Some candidates did not simplify the sequence, leaving it as $\mathrm{T}_n = 4 + (n - 1)6$ instead of $6n - 2$ as the final answer.

Answers: $u_{r+1} - u_r = 6r^2$; $u_n = 6n - 2$; 2020000

Question 3

Matrix question is a favourite topic which comes out almost every year. The question this year consists of two unknowns as entries in the matrix resulting in low performance in this topic. Candidates were unable to identify the properties of an upper triangle matrix. Almost all the candidates were able to find the $\mathbf{A}^2$. Quite a number of candidates knew what upper triangular matrix is and were able to deduce the equation $p + pq = 0$ in terms of p and q successfully. There were candidates that could not proceed after finding $\mathbf{A}^2$ since they did not know what the upper triangular matrix is. They confused it with lower triangular matrix or symmetrical matrix and some thought that the elements along the major diagonal equals to zero. Most of the candidates who were able to proceed with $p + pq = 0$, were unable to solve the equation correctly. Majority of candidates showed $p + pq = 0 \Rightarrow pq = 1 - p$, $\therefore\ q = -1$. For candidates who proceed with $\det(\mathbf{A}^2) = 81$, most of them used the long way by expanding along the first row to get the determinant, but did not know how to continue. Some continued by expanding the equation and made a lot of careless mistakes and then just gave up. A few candidates who managed to get $g^4 = 81$ or $(p + 1)^4 = 81$ only gave one value where $q = 3$ or $p + 1 = 3$. A few candidates find $\mathbf{A}^2$ using wrong multiplication where they squared all elements in matrix $\mathbf{A}$, $(a_{ij})^2$. Candidates were unable to perform the multiplication of a matrix. Surprisingly, a few candidates even multiplied each entry to itself.

Answers: $\begin{pmatrix} 1 + p & 1 + q & -2 - 2p + 2q \\ p + pq & p + q^2 & -2p \\ 0 & 0 & (p - q)^2 \end{pmatrix}$

Question 4

Most candidates managed to attain the first four marks but quite a number of candidates got '*no essential working*' deduction mark for not presenting the working of the answer. Candidates wrote $\dfrac{z}{w} = \sqrt{2}\left(\cos\left(-\dfrac{7\pi}{12}\right) + i\ \sin\left(-\dfrac{7\pi}{12}\right)\right)$ without showing the important steps. Generally, candidates who made an effort to answer this question scored quite good marks. Candidates understood the two forms of a complex number; polar and Cartesian. Quite a number of candidates were able to find $\dfrac{z}{w}$ in polar form knowing the fact that $\left|\dfrac{z}{w}\right| = \dfrac{|z|}{|w|}$ and $arg\left(\dfrac{z}{w}\right) = arg(z) - arg(w)$. The candidates were able to use de Moivre's theorem and to find correctly $\dfrac{\sqrt{8}}{2}$ and $\dfrac{\pi}{4} - \dfrac{5\pi}{6}$. Quite a number of candidates were able to change from polar to Cartesian form and then proceeded to divide the two complex numbers by multiplying the conjugate of w to find $\dfrac{z}{w}$ in Cartesian form. For candidates who proceed until the end, most of them were able to compare real part of $\dfrac{z}{w}$ of polar form and Cartesian form and using the reason cos (–) = cos (+) to deduce the result. Quite a number of candidates did not show clear workings as to how the answer was obtained. These candidates used the scientific calculator to expressed w and z in Cartesian form directly from the calculator. Some candidates also found $\dfrac{z}{w}$ directly from the calculator without clear step-by-step working. Some candidates did not provide an exact answer but expressed $\dfrac{z}{w}$ in decimal form. For candidates who used modulus and argument of w and z to find Cartesian form, a majority of the candidates could not select the x-value correctly. Most of them just took the positive value without looking at which quadrant it lied. A few candidates did not simplify the value and left the answer as $\frac{-2\sqrt{3}+2}{4} - \frac{2\sqrt{3}+2}{4}\mathrm{i}$ or $\frac{-\sqrt{3}+1-(\sqrt{3}+1)\mathrm{i}}{2}$. Most of the candidates did not give the reason of how $\cos\left(-\frac{7\pi}{12}\right) = \frac{1-\sqrt{3}}{2\sqrt{2}}$ became $\cos\left(\frac{7\pi}{12}\right) = \frac{1-\sqrt{3}}{2\sqrt{2}}$.

Answers: (*a*) $\frac{z}{w} = \sqrt{2}\left(\cos\left(-\frac{7\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{7\pi}{12}\right)\right)$;

(*b*) $w = -\sqrt{3} + \mathrm{i}$ and $z = 2 + 2\mathrm{i}$;

(*c*) $\frac{z}{w} = \frac{1-\sqrt{3}}{2} - \frac{1+\sqrt{3}}{2}\mathrm{i}$

Question 5

Performance of the candidates for this question was very poor. Most candidates attempted this task but failed to get good marks. Quite a number of candidates were able to determine the coordinate of the centre using the two equations of the asymptotes given. Some candidates were able to use distance between vertex and centre to find $b = 3 - (-3) = 6$, and used the gradient formula $\frac{6}{a} = 3 \Rightarrow a = 2$ to find the value of a which led to writing the hyperbola equation. Some candidates did not know how to go about determining the correct form of the hyperbola equation and obtained the wrong form of the equation, which they wrote $\frac{(x+2)^2}{a^2} - \frac{(y-3)^2}{b^2} = 1$. Some candidates managed to find the intersection point between the two asymptotes but thought that it was coordinate of another vertex. Quite a number of candidates made careless mistakes when finding the values of a and b, and some gave negative values. Most candidates were not able to sketch the shape of the hyperbola. The curve is far from asymptotes and some even curved back. Quite a number of candidates were not able to answer this question thus not able to score any marks because they failed to obtain the values of a and b correctly as they do not know that the point of intersection of the asymptotes gave the centre of the hyperbola. A handful of the candidates were able to find the coordinates of the centre and vertices correctly but they failed to sketch the correct shape of the hyperbola.

Answers: (*a*) $\frac{(y-3)^2}{6^2} - \frac{(x+2)^2}{3^2} = 1$, $(-2, 9)$

Question 6

This question was poorly attempted by most candidates and it became one of the questions that has a high number of non-attempts. Only a few candidates who attempted scored full marks for this question. Performance of the candidates in this question were poor. Only some candidates managed to use the concept of unit vector to obtain the two values of λ. By using the concepts of parallel and perpendicular, candidates were able to use the dot product $\overrightarrow{BA} \cdot \overrightarrow{BC} = 0$ to attain the values of μ. To obtain the answer, candidates used $\begin{pmatrix} -1 \\ -\lambda \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ \mu \end{pmatrix} = 0$ or $\begin{pmatrix} 1 \\ \lambda \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -1 \\ -\mu \end{pmatrix} = 0$ to proceed correctly. Most candidates were not able to obtain values of λ since they did not use the fact that the length of a unit vector is always one. Some candidates managed to get $\lambda^2 = 4$ but weak in solving quadratic equation, where they solved the case for $\lambda = 2$ only. Many candidates used $\begin{pmatrix} 1 \\ \lambda \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ \mu \end{pmatrix} = 0$ to etermine the relationship between λ and μ, which was incorrect since the candidates did not consider the direction of the vectors. They failed to relate the concept of a parallel vector as a scalar of a vector by just assuming that $\overrightarrow{AB} = \frac{1}{3}(\mathbf{i} + \lambda\mathbf{j} + 2\mathbf{k})$ and $\overrightarrow{BC} = 4\mathbf{i} + \mathbf{j} + \mu\mathbf{k}$ which were supposed to be $\overrightarrow{AB} = \frac{h}{3}(\mathbf{i} + \lambda\mathbf{j} + 2\mathbf{k})$ and $\overrightarrow{BC} = k(4\mathbf{i} + \mathbf{j} + \mu\mathbf{k})$ respectively, whereby h and k are scalars. Some failed to apply the correct definition of angle between two vectors. Quite a number of candidates wrote irrelevant workings which was not related to the question, for example $\overrightarrow{AB} \times \overrightarrow{BC}$.

Answers: $\lambda = -2, 2,\ \ \mu = -3, -1$

Question 7

Most candidates who attempted this question were able to score full marks in part (*b*). They generally demonstrated the ability to construct the correct augmented matrix and apply Elementary Row Operations (ERO) to reduce it to row-echelon form, although some made minor calculation errors. A significant number correctly equated the entries (3,3) = 0 and (3,4) = 0, and solved the resulting equations to find values of k. In part (*b*)(iii), many candidates successfully equated entry (3,3) multiplied by (−1) with entry (3,4), obtained two possible values of k, and provided the correct reasoning to reject the invalid one. Many were also able to determine the case of infinitely many solutions by letting $z = tz$ and expressing x and y in terms of the parameter t. However, some candidates were confused between a matrix equation and an augmented matrix. Many did not understand the conditions for the system to have infinitely many solutions, where both (3,3) = 0 and (3,4) = 0, and for no solution, where (3,3) = 0 and (3,4) ≠ 0. Consequently, they failed to conclude with the correct intersection concept ("$\cap$"). Some candidates listed their answers without proper conclusions, simply stating: (i) $k = -2, 2$ and $k = 2$ (ii) $k = -2, 2$ and $k \neq 2$. Moreover, several who equated (3,3) × (−1) with (3,4) did not reject $k = 2k$ as required. Others avoided using the REF to do so, instead relying on the third original equation, which was incorrect. A number of candidates made errors when solving quadratic equations. A few weaker candidates lacked understanding of augmented matrices and row-echelon form altogether, and mistakenly attempted to find the inverse of the matrix using ERO. Some did not use appropriate symbols for ERO, and there was inconsistency in using short and long arrows. Improper ERO notations, such as using equal signs instead of arrows, were also observed.

Answers: (a) $\begin{pmatrix} 3 & -1 & 5 \\ 1 & 2 & -3 \\ 4 & 1 & k^2-2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ k+3 \end{pmatrix}$; (*b*) $\left(\begin{array}{ccc|c} 3 & -1 & 5 & 2 \\ 0 & 7 & -14 & 7 \\ 0 & 0 & 3k^2-12 & 3k-6 \end{array}\right)$

(b) (i) $k = 2$; (ii) $k = -2$; (iii) $k = -3$; For $k = 2 \Rightarrow x = 1 - t$; $y = 1 + 2t$; $z = t$

Question 8

The question appeared to be a tough question for the candidates. Only a small number attempted this question and most of them could not fully fulfil the whole part of the question. Parts (*a*) and (*b*) were straight forward but a majority of those who attempted this question did not do well in part (c). Candidates that attempted this question were able to equate the two lines and formed three linear equations in terms of λ and μ. For part (*b*), many candidates successfully performed the cross product to get the normal vector $\begin{pmatrix}0\\6\\3\end{pmatrix}$. They were able to choose an appropriate point and formed the Cartesian equation of the plane, $6y + 3z = 21$ or $2y + z = 7$ correctly. However, there were a number of candidates that did not show proper working because they used calculators. For part (*c*)(i), quite a number of candidates were able to find the direction of the line using cross product correctly. A few of them were able to solve the simultaneous equations of Π_1 and Π_2 to get the common point and managed to form the vector equation of line of intersection. Candidates used two equations to obtain the values of λ and μ, and straight away determine the coordinate of point of intersection (3, 0, 7) without verifying that these values also satisfied the 3rd equation. Some candidates were penalised in writing the coordinate of the point of intersection in the form of $\begin{pmatrix}3\\0\\7\end{pmatrix}$, which was not in the coordinate form. A few candidates forgot to conclude that the lines were intersected after obtaining λ and μ, and checking the consistency. For part (*b*), there were candidates that did not understand what is the Cartesian equation of a plane, and instead gave their final answer in vector form, $\mathbf{r}\begin{pmatrix}3\\0\\7\end{pmatrix} = 21$. Some candidates made mistakes by using position vector of points of ℓ_1, $\begin{pmatrix}1\\2\\3\end{pmatrix}$ and ℓ_2, $\begin{pmatrix}5\\1\\5\end{pmatrix}$ in cross product to get the normal of the plane. For part (*c*)(i), some candidates did not solve the simultaneous equations of Π_1 and Π_2 to get the common point. They simply used (3, 0, 7) as a common point. Some candidates failed to show working in obtaining normal vector and dot product, the candidates were penalised as '*no essential working*'. Quite a number of candidates were penalised because of the way of writing the vector equation of line, for example $\ell : (-2\mathbf{i} + 7\mathbf{k}) + \lambda(2\mathbf{j} - 4\mathbf{k})$. Very few candidates attempted part (*c*)(ii). Those who attempted this part were unable to get the correct answer because they did not understand the requirement of the question and used the wrong method to solve it. Most candidates who attempted this question avoided in answering this part of the question except for a few candidates who were able to attained the equation of the plane, Π_1 in part (*b*).

Answers: (*b*) $\Pi_1 = 2y + z = 7$;

(*c*) (i) $\mathbf{r} = \mathbf{r} = \begin{pmatrix}-2\\0\\7\end{pmatrix} + t\begin{pmatrix}0\\-1\\2\end{pmatrix}$, for $t \in$; (ii) $p = 4$

Mathematics (T) 954/2

OVERALL PERFORMANCE

The number of candidates for this subject was 2 554. The percentage of candidates who obtained a full pass was 51.41%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	9.29	5.00	6.34	6.26	7.32	7.28	9.92	4.19	2.99	4.57	36.84

RESPONSES OF CANDIDATES

PAPER 954/2

General comments

The examination paper provided a fair opportunity for candidates to demonstrate their understanding of mathematical concepts and their ability to apply problem-solving skills. Candidates who were well prepared generally performed very well. These good candidates showed clear planning and presented their answers in an organised manner, with all essential workings included. Their handwriting was neat and legible, and they were able to sketch graphs accurately with correct labels. Additionally, they provided relevant and appropriate explanations in response to the questions.

Candidates in the moderate range were able to respond reasonably well to questions they were familiar with, often managing to correctly answer the initial parts. However, they struggled with questions that required higher-order thinking or deeper conceptual understanding. For instance, Question 8, which required understanding of half-life, and Question 4, which involved solving a modified form of a linear ordinary differential equation, proved difficult for this group.

Weaker candidates demonstrated poor grasp of basic mathematical concepts and had difficulty with algebraic manipulation. Their responses were often disorganised and did not meet the requirements of the questions. Many of these candidates showed limited understanding, were unsure of the concepts involved, or were unable to recall or apply even the formulae provided in the question booklet. A few continued the practice of dividing pages into two columns, which made it difficult for examiners to assess their answers clearly and should be strongly discouraged.

In general, many strong scripts were observed, and the quality of presentation was usually good. The paper allowed well-prepared students to showcase their knowledge and understanding effectively. However, the poor performance of some candidates indicated a lack of preparation, as seen in Question 3, where trigonometric substitution was required. Despite the question being straightforward, many students used an incorrect method, namely integration by parts, which could not solve the problem.

In Section A, Questions 1, 2 and 6 were generally well answered, while Question 3 was poorly done due to the widespread use of the wrong method. Questions 4, 5, 7 and 8 were answered with moderate success. A notable issue in Question 4 was that many candidates were unable to use the integrating factor technique correctly. However, performance in Question 5, which tested understanding of the Maclaurin series, showed improvement compared to previous years.

In Section B, the majority of candidates chose to answer Question 7, which involved applying integration to find area and volume. This question was attempted with varying levels of success. In contrast, only a small number of candidates chose Question 8, which involved forming a differential equation to model a real-life situation of drinking coffee. Most of those who attempted it failed to sketch the required graph correctly.

Comments on individual questions

Question 1

Students were required to determine the value of f(4), the left-hand limit (LHL), and the right-hand limit (RHL). By equating the RHL to the LHL, they could solve for n, and by setting f(4) equal to the LHL, they could solve for m. This process involves applying the concept of continuity of a function at a point. Many candidates successfully identified the correct expressions for both the LHL and RHL, and used the proper limit notation, including knowing when to omit it. They were generally able to equate RHL to LHL and attempted to solve for n. However, a significant number of students struggled with factorisation of $64 - x^3$ or simplification of $\frac{64 - x^3}{x - 4}$, leading to errors. Common mistakes included incorrect factorisation steps; i.e $\lim_{x\to4^-}\frac{64 - x^3}{x - 4} = \lim_{x\to4^-}\frac{(4 - x)(4 - x)(4 - x)}{x - 4}$ and incorrect cancellation; i.e $\lim_{x\to4^-}\frac{64 - x^3}{x - 4} = \lim_{x\to4^-}\frac{(x - 4)(x^2 + 4x + 16)}{x - 4} = \lim_{x\to4^-}(x^2 + 4x + 16)$, which resulted in incorrect evaluation. Some candidates incorrectly considered only the positive value of m, ignoring the negative solution. Very few students applied L'Hôpital's rule to evaluate the limit. A number of students mistakenly used the expression for the limit as the value of f(4), which prevented them from correctly solving for m. Additionally, many students only gave $m = 2$ rather than considering both possible values, $m = \pm2$.

Answers: $m = -2, 2$, $n = -16\sqrt{3}$

Question 2

Candidates were generally able to differentiate both x and y with respect to t, correctly applying the quotient or product rule as needed, along with the appropriate use of the chain rule to find $\frac{dy}{dx}$. Most were able to form and solve a simple quadratic equation in terms of t, successfully obtaining two values. These values were then accurately substituted back into the given parametric equations to find the corresponding points on the curve, correctly identifying them as (0,1) and (0,5). In general, candidates understood that when the tangent to the curve is parallel to the x-axis, they needed to equate $\frac{dy}{dx} = 0$. For part (*b*), a fair number of candidates demonstrated awareness that a tangent line parallel to the y-axis implies an infinite gradient, which occurs when the denominator of $\frac{dy}{dx}$ is zero. However, most concluded that there is no tangent line to the curve that is parallel to the y-axis. Nonetheless, some candidates exhibited weaknesses in algebraic manipulation, which affected the accuracy of their final answers. However, algebraic weaknesses were seen in some candidates' work such as:

$\frac{dx}{dt} = \frac{3t(18t) - (9t^2 - 1)(3)}{(3t)^2} = \frac{54t^2 - 27t^2 - 3}{9t^2}$, $\frac{dy}{dt} = \frac{3t(18t + 9) - (9t^2 - 9t)(3)}{(3t)^2} = \frac{54t^2 + 27t - 27t + 3}{9t^2}$ and $\frac{dx}{dt} = \frac{3t(18t) - (9t^2 - 1)(3)}{(3t)^2}$. These mistakes led to the wrong answer of $\frac{dy}{dx}$ and difficult for them to proceed. Some candidates treated x and y as products of two functions and applied the product rule to obtain $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Among these, a few repeated a common mistake by incorrectly writing $(3t)^{-1}$ as $3t^{-1}$. Surprisingly, a significant number of candidates misunderstood the concept of a tangent line parallel to the x-axis, incorrectly stating that it implies $x = 0$. For part (*b*), many candidates failed to provide a correct or sufficiently detailed explanation for why there is no tangent line to the curve that is parallel to the y-axis. Some managed to state the correct reasoning but did not follow through with a complete conclusion. There were also several candidates who could not apply the quotient rule properly, despite the fact that it was included in the mathematical formulae sheet provided in the examination paper. When solving for values of t, many candidates were unable to find the complete set of real solutions. In several cases, they omitted the negative value of t, which led to identifying only one point instead of both required points. Most candidates struggled with part (*b*) primarily because they did not understand what it means for a tangent line to be parallel to the y-axis. This lack of conceptual understanding resulted in many responses without any justification or with incorrect reasoning, for which no marks could be awarded.

Answers: (0, 1) and (0, 5)

Question 3

The majority of the candidates could not answer this question correctly. Most of them tried to start by performing part by part integration and therefore they wrote messy and meaningless work. Only a few candidates were able to write in the form of $\int \frac{f'(x)}{f(x)} dx$ or use correct substitution to obtain the integral. Candidates who use the correct trigonometry identity $\cot(2e^x) = \frac{\cos 2e^x}{\sin 2e^x}$ and using the formula $\int \frac{f'(x)}{f(x)} dx$ or using correct substitution method can solve this question. However, among these candidates, mistakes also occurred such as $\int \frac{e^x \cos 2e^x}{\sin 2e^x} dx = \text{In}[\sin(2e^x)]$ and $\int \frac{\cos 2u}{\sin 2u} du = \text{In}[\sin(2u)]$.

Answers: $-\frac{1}{2}\text{In}(\sin 2)$

Question 4

Many candidates recognised that the given differential equation was a first-order linear differential equation and that they needed to find the integrating factor. Their strengths were obviously seen in writing the differential equation in the standard form of first order linear ODE, determining the integrating factor, and following the correct steps up to the point where the equation was ready to be integrated. However, most candidates did not know how to carry out the $\int x^3 e^{-x^4} dx$. Similar to question 3, many attempted to use integration by parts, which led to incorrect results. Only a small number of candidates managed to express the equation in the correct form and successfully performed $\int f'(x)e^{f(x)} dx$ to arrive at the correct answer.

Answers: $y = \frac{1}{4x^5}\left(\frac{1}{e}, \frac{1}{e^{x^4}}\right)$

Question 5

Most of the candidates were able to use Maclaurin series of $\cos x$ to obtain Maclaurin series of $\cos 2x$ correctly. Only a few candidates couldn't get it right where they wrote $\cos 2x = 1 - \frac{2x^2}{2!} + \frac{2x^4}{4!} + \dots$. Candidates tends to make careless mistake when multiply series of e^x with series of $1 + \cos 2x$ and therefore could not get marks for correct series obtained. For evaluating the limit, quite a number of candidates used incorrect trigo identity which they used $\cos 2x = \cos^2 x - 1$ and therefore could not evaluate the limit correctly. Some candidates did not follow the requirement of the questions. They did not use their series obtained earlier to proceed. They tried to find series of $e^x \cos^2 x$ by multiplying series of e^x with series of $\cos x \times \cos x$.

Answers: $e^x(1 + \cos 2x) = 2 + 2x - x^2 - \frac{5x^3}{3} - \frac{x^4}{4} + \dots$; 1

Question 6

Quite a number of candidates did not fully grasp the requirements of the question. A common error in the first part was incorrectly defining the function and then attempting to show that f(3) and f(4) had opposite signs to conclude that a root existed within the interval [3,4], without proper justification. Some used the wrong function altogether, while others provided incomplete conclusions, such as failing to mention the interval or clearly state the necessary condition. Even among those who identified the correct function, many were unable to differentiate it correctly. In the second part, most candidates were able to carry out the iterative process correctly and obtain an approximate root. However, a few misunderstood the question and attempted to apply the Newton-Raphson method, which led them to incorrect or no solutions.

Answers: Root = 3.391 (3 dp)

Question 7

For part (*a*), most of the candidates were able to state coordinates of P correctly. However, a few of them tried to find intersection point between two curves and could not solve the equation. A few candidates wrote coordinate y in decimal form. For part (*b*), many candidates could not define the required area correctly. Some used the wrong limit and some used lower curve–upper curve. For the integration parts, most of the candidates were able to integrate $\int\left[\left(x - \frac{1}{2}\right)^2 + \frac{\pi}{4}\right]\mathrm{d}x$ or $\int\left(\frac{1}{2} + \sqrt{y - \frac{\pi}{4}}\right)\mathrm{d}y$ but they could not integrate $\int \tan^{-1}(2x)\mathrm{d}x$ or $\int \frac{1}{2}\tan^{-1}(y)\mathrm{d}y$ correctly. For part (*c*), many candidates did not attempt to answer this part. Only good candidates were able to define the required volume correctly. Many candidates defined volume wrongly. Common mistakes were found such as: $V = \pi\int \frac{\tan y}{2}\mathrm{d}y$, $V = \int_{\frac{x}{4}}^{1.144}\left(\frac{\tan y}{2}\right)^2 \mathrm{d}y$, $V = \pi\int[\tan^{-1}(2x)]^2\mathrm{d}x$, wrong trigonometry identity used for $\tan^2 y$, and did not leave answer in exact form as required. Candidates tried to solve the equations of the two curves given. Unfortunately, candidates were unable to find the integration $\int \tan^{-1}(2x)\mathrm{d}x$ by using integration by part and followed by the formula $\int \frac{f'(x)}{f(x)}\mathrm{d}x = \text{In}|f(x)| + c$ due to weak in integration of trigonometric functions. Candidates were also unable to find the volume by using the integration because they were weak in trigonometry identity. Therefore, they could not proceed finding the integration. In addition, some candidates struggled to handle limits involving odd decimal values such as 1.099 or 1.144, especially when these appeared within trigonometric functions.

Answers: (*a*) $\left(\frac{1}{2}, \frac{\pi}{4}\right)$; (*b*) 0.054753; (*c*) $\frac{\pi}{4}\left(1 - \frac{\pi}{4}\right)$

Question 8

Candidates who chose this question generally performed well, as many were able to formulate the correct differential equation. They demonstrated strong skills in solving differential equations and showed a clear understanding of the given conditions in each part of the question. In part (*a*)(i), high-performing candidates correctly derived the differential equation $\frac{dx}{dt} = -kx$ and applied the method of separation of variables appropriately. They integrated both sides accurately and arrived at the correct general solution. Furthermore, they successfully substituted $t = 5$ and $x = \frac{1}{2}x_0$ into the equation from part (*a*)(i) to determine the correct value of $k = \frac{\text{In}2}{5}$. In the subsequent parts, most candidates were able to sketch the correct shape of the graph, showing that when $t = 0$ and $x = x_0$, and the curve asymptotically approaches zero as time progresses towards infinity. For part (*b*)(i), many candidates correctly substituted $x_0 = 95$ and $t = 8$ to find the value of x. Then, they substituted $x = \frac{2}{100} \times 95$ and got the correct value of t. A small number of candidates were also able to justify, using the concept of a limiting value, that caffeine cannot be completely eliminated from the body.

Among the moderate candidates, some struggled to derive the correct differential equation and instead used incorrect forms, $\frac{dx}{dt} = kx$, which led to errors throughout their solutions. A number of them showed misunderstanding of the concept of half-life, resulting in incorrect values for the constant k. Some assumed the initial mass of caffeine, x, was 1.0 and concluded it halved to 0.5 after five hours, which was not necessarily applicable. Additionally, a few candidates failed to label their graphs properly. In part (*b*), the moderate candidates often substituted the correct values of $x_0 = 95$ and $t = 8$, but made calculation errors to get the value of t. Some incorrectly used values such as $x = 0.02$ and $x_0 = 1.0$ instead of the correct $x = 1.9$ mg, which led to inaccurate conclusions about caffeine elimination. Their reasoning was often flawed or inadequately explained.

At the lower end of the performance spectrum, weak candidates were unable to formulate a valid differential equation and had no clear strategy to tackle the question. As a result, many left it blank and did not earn any marks.

Answers: (*a*) (i) $\frac{dx}{dt} = -kx$, $x = x_0e^{-kt}$,

(ii) $k = \frac{\text{In}2}{5}$;

(*b*) (i) $x = 31.338$ mg,

(ii) $t = 28.219$ hours, amount of caffeine cannot totally be eliminated from the body.

Mathematics (T) 954/3

OVERALL PERFORMANCE

The number of candidates for this subject was 2 536. The percentage of candidates who obtained a full pass was 61.40%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	7.53	5.84	7.81	8.68	9.63	11.72	10.19	4.13	4.42	5.01	25.04

RESPONSES OF CANDIDATES

PAPER 954/3

General comments

In general, candidates demonstrated a broad spectrum of mathematical abilities, resulting in answers of varying quality. Their responses reflected a wide range of understanding of the statistical concepts assessed in the paper. Some candidates approached the questions methodically and presented well-structured answers, indicating a clear grasp of both the questions and underlying concepts. All candidates responded in English. Overall, their command of the language was satisfactory, with only occasional grammatical errors that did not significantly hinder comprehension.

High-performing candidates displayed a strong understanding of statistical concepts and produced well-organised, precise responses. Their solutions were clearly planned and presented neatly, with all essential workings shown in a systematic and accurate manner. They used correct mathematical symbols consistently and provided appropriate explanations or descriptions when required. Their responses reflected clarity of thought and confidence in applying the concepts.

Moderate candidates were generally able to answer the parts of the questions they were familiar with. Many showed partial understanding and could complete the initial steps of a question, though some solutions contained errors, particularly in the use of symbols or mathematical language. While many of their attempts were reasonable, a number of responses lacked completeness or accuracy in the final steps.

Low-performing candidates struggled due to insufficient understanding of basic statistical principles. Their responses often failed to address the questions as required, and they were generally unable to apply the necessary mathematical concepts to solve the problems. Answers were often disorganised and poorly presented, with many questions either left blank or only partially attempted. Their work revealed significant weaknesses, such as misunderstanding the questions, unfamiliarity with key concepts, forgetting relevant formulas, or being unable to apply formulas even when provided in the question paper or booklet.

Comments on individual questions

Question 1

Most of the candidates were able to draw the correct shapes of six bars of histogram with correct scale for x-axis and y-axis using class boundaries or class mid-points with equivalent scale and labelled the axes correctly. Unfortunately, some candidates made errors in determining the correct bar heights or left unnecessary gaps between bars when plotting the histogram. A small number of candidates draw a boxand-whisker plot and ogive instead of histogram, made them lose all the related marks that followed. They were able to state the distribution of the blood glucose levels as positively skewed or skewed to the right distribution. Only about half of the candidates were able to obtain the mode by using the interpolation method clearly on the second bar of the histogram. The rest of candidates either used the formula to find the mode, or only stated the modal class, 6.0 – 7.0, without the value of the mode.

Majority of the candidates were able to obtain the correct mean for the data by showing all the working correctly. However, a few of the candidates just stated the value of mean $\frac{729}{100}$ or 7.29 without any working. Most candidates were able to apply the measure of central tendency (mode and mean) to justify the skewness of the distribution. They were able to show that the mean obtained in (*b*) is greater than the value of mode obtained in (*a*)(ii) and verify their answer in (*a*)(i). A small number of candidates even used Pearson's coefficient of skewness to support their conclusion. Nevertheless, a few candidates misunderstood the requirement of the question and failed to use the obtained values of mode and mean to justify the type of distribution, resulting in incomplete or incorrect answers in part (*c*).

Answers: (*a*)(ii) 6.75 ; (*b*) 7.29 ; (*c*) Mean $>$ Mode

Question 2

Overall, the performance for this question was very poor. Candidates who correctly constructed a tree diagram based on the given situation were generally able to solve the problem with ease, compared to those who did not. The majority of those who did not draw the tree diagram appeared to struggle due to a lack of understanding of the question.

In part (*a*), some candidates failed to recognise that there are two possible ways for Player A to win: one where Player A makes the first move and wins, and another where Player B makes the first move but Player A still ends up winning. As a result, they gave only $0.5k$ as the answer, instead of the correct expression $0.5k + (1 - k)(0.3)$. A similar mistake was observed when calculating the probability of Player B winning.

In part (*b*), many candidates correctly identified that the probability for a fair coin is 0.5, allowing them to reach the correct answer. A few candidates also applied the Law of Total Probability appropriately in their solutions. However, the use of conditional probability in both parts (*a*) and (*b*) was limited, with only a small number of candidates demonstrating a clear understanding of how to apply it effectively.

Answers: (*a*) (i) $0.3 + 0.2k$; (ii) $\frac{3}{7}$ or 0.42857; (*b*) $\frac{1}{4}$ or 0.25

Question 3

In part (a) of the question, most candidates were able to find the value of λ and correctly apply the Poisson formula to determine the required probability. This was because they understood the properties of the Poisson distribution, specifically that $\lambda = E(X) = Var(X)$. They correctly used the identity $Var(X) = E(X^2) - (E(X))^2$. A small percentage of candidates made various errors in their responses. Some of them incorrectly assumed $\lambda^2 = 24$, revealing a misunderstanding of the variance formula. Others formed an incorrect quadratic equation such as $\lambda^2 - \lambda + 42 = 0$, that could not be solved, indicating confusion in algebraic manipulation. There were also candidates who managed to solve the quadratic equation correctly and obtained two values $\lambda = -7$ and $\lambda = 6$, but failed to reject the negative one, which is invalid in the context of a Poisson distribution. Additionally, some candidates applied an entirely incorrect formula, such as $\lambda = 24 - \lambda$, which led to an incorrect final answer of $\lambda = 21$.These mistakes reflect gaps in algebraic manipulation and conceptual understanding of the Poisson distribution's properties.

In part (b), the majority of candidates demonstrated a good understanding of using the complement rule to find the required probability. Many correctly expressed $(X \geqslant 3) = 1 - (X < 3)$ or $1 - [P(X = 0) + P(X = 1) + P(X = 2)]$ and were able to apply the correct Poisson formula using their value of λ from part (a) to obtain the final answer. However, some candidates made errors in their working. One common mistake was writing $(X \geqslant 3) = 1 - (X \leqslant 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)]$, which led to an underestimated value for the probability. Others attempted to calculate $(X \geqslant 3) = (X = 4) + P(X = 5) + P(X = 6) + \ldots$, which is correct in theory but inefficient and impractical without technology or a table, especially since it may not lead to an accurate answer manually. Some candidates used both values of λ obtained in part (a), including the invalid negative value $\lambda = -7$, and worked out two sets of probabilities, which showed a failure to reject the inadmissible solution. Additionally, a number of candidates applied the wrong formula for the Poisson distribution, such as incorrect substitution into $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, leading to computational errors. These mistakes reflect misunderstandings in the proper use of probability laws, incorrect handling of solutions from earlier parts, or procedural errors in applying the Poisson distribution formula.

Answers: (a) $\lambda = 6$; (b) 0.93803

Question 4

Almost all candidates were able to estimate the proportion of the processed food products as $\dfrac{66}{400}$ and simplified it to $\dfrac{33}{200}$ or 0.165. However, a notable number of candidates did not use the appropriate notation for proportion, such as p_s or $\hat{p}$. Most candidates who attempted this question were able to find the standard error, $\sqrt{\dfrac{0.165 \times 0.835}{n}}$ but some failed to form an inequality because they were not able to interpret the term "within 0.02" correctly. There were some who used the sign "$<$" or "=" instead of "$\leqslant$". Quite a number of candidates were unable to proceed to find the value of n using the correct method because they were not proficient in solving inequalities involving square roots sign. Some did not even show proper working to find n, and quite a number did not write the word "smallest" explicitly, instead leaving the answer as $n = 933$. A small percentage of students used the concept of probability to obtain the value of n as:

$$P(-0.20 \leqslant p_s - p \leqslant 0.20) \geqslant 0.90 \Rightarrow P\left(\frac{0.02}{\sqrt{\frac{0.165 \times 0.835}{n}}}\right) \leqslant Z \geqslant \left(\frac{0.02}{\sqrt{\frac{0.165 \times 0.835}{n}}}\right) \geqslant 0.90.$$

Most of the candidates simply wrote "sample size increases" without any justification of the effect of increasing the confidence level (k) on the value of $Z_{\frac{\alpha}{2}}$. This is likely because this type of question is common and students are familiar with it. A few candidates justified their answer by calculating the value of n for $k = 96\%$ as 1454 and were able to compare their answer with the case when $k = 90\%$.

Answers: (*a*) $\frac{33}{200}$ or 0.165; (*b*)(i) 933; (ii) Sample size increases

Question 5

Many candidates were able to correctly state the hypotheses, apply the appropriate test statistic formula, and compare their test statistic with the critical value to draw the correct conclusion. However, a substantial number of candidates mistakenly assumed that 50 N m^{-2} was the standard deviation of the given population. As a result, they incorrectly calculated the standard error as $\sqrt{\frac{50^2}{40}}$ instead of the correct expression $\sqrt{\frac{\frac{40}{39} \times 50^2}{40}}$. Consequently, they obtained an incorrect test statistic, $Z = \frac{980 - 1000}{\sqrt{\frac{50^2}{40}}}$

Several common mistakes were observed during the test. Some candidates used an incorrect symbol for the alternative hypothesis H_1, such as "$>$". Others stated the hypotheses using the notation $\bar{x}$ or p instead of using the population mean symbol μ. There were also candidates who wrote the hypotheses incorrectly, for example stating H_0: $\mu = 980$ and H_1: $\mu < 980$, which does not reflect the correct alternative. In addition, some candidates provided the wrong critical values, such as $z_\alpha = 2.326$ or $z_\alpha = 2.054$. Finally, a number of candidates made conclusions without referring to the 1% significance level or failed to include appropriate phrases such as 'sufficient evidence' or 'insufficient evidence'.

Answers: –

Question 6

Since this is a straightforward and frequently asked question, even candidates of average ability were generally able to answer it well. Most candidates successfully performed the test in a correct manner and demonstrated an understanding of the purpose of the goodness-of-fit test. The majority were able to state the correct hypotheses, calculate the corresponding probabilities and expected values, and, based on the expected frequencies (E_i), compute the test statistic accurately. They then compared the test statistic with the critical value obtained earlier, made a decision, and arrived at a correct conclusion. However, several noticeable mistakes were observed among weaker candidates. Some of them interchanged the null hypothesis (H_0) and the alternative hypothesis (H_1), while others failed to calculate the required probabilities correctly; in some cases, they were unable to compute the probabilities at all. Although a number of these candidates correctly calculated the expected frequencies for each cell, they mistakenly combined the observed frequencies of cells 1 and 2, and of cells 4 and 5. This led to an incorrect determination of the degrees of freedom, which subsequently resulted in the use of an incorrect critical value and test statistic. Additionally, some candidates wrote conclusions without referring to the "1% significance level." or without using the appropriate phrases such as "sufficient evidence" or "insufficient evidence." Another common issue was the inconsistent rounding of values. Candidates presented answers using 3, 4, or 5 significant figures but failed to maintain a consistent level of accuracy throughout their calculations.

Answers: –

Question 7

Fewer candidates chose to answer this question compared to Question 8, even though it was relatively straightforward and easy to attempt. This could be attributed to the involvement of calculus, which many candidates find challenging, as well as the presence of multiple unknowns in the given probability density function, which increased the complexity of the problem. Candidates who answered part (*a*) correctly generally demonstrated an understanding of how to find the values of m and h. However, only a small number of candidates attempted and successfully completed the entire question.

In part (*a*), most candidates successfully applied the property of the probability density function $\int_0^4 hx(m - x)\mathrm{d}x = 1$. They carried out the integration correctly, substituted the limits properly, and showed all necessary steps to establish a relationship between h and m. However, a few candidates struggled with the integration process, and only a small number opted to use the cumulative frequency method to derive the relation accurately.

For part (*b*)(i), less than half of the candidates correctly defined the expected value $E(X)$ as $E(X) = \int_0^4 x(hx(m - x))\mathrm{d}x = \left[hm\frac{x^3}{3} - h\frac{x^4}{4}\right]_0^4$. Many were able to isolate h in terms of m as $h = \frac{7.2}{64m - 192}$ although some minor miscalculations were observed.

Part (*b*)(ii) showed generally poor performance. A significant number of candidates failed to correctly interpret the structure of the taxi fare, which included a fixed base charge and a consistent per-kilometer rate. Only about 10% formed the correct linear relationship for the fare, Y, in terms of the distance, X, expressed as either $Y = 3 + 5XY = 3 + 5X$ or $Y = 0.3 + 0.5XY = 0.3 + 0.5X$. Those who did were able to compute the expected fare precisely, such as $E(Y) = 3 + 5(2.4)$ or $E(Y) = 0.3 + 0.5(2.4)$.

In part (*b*)(iii), candidate performance was similarly weak. Only a small fraction successfully calculated the probability that the fare exceeded 13, using the relationship $P(Y > 13) = P(3 + 5X > 13) = P(X > 2)$.

These candidates correctly evaluated $P(X > 2) = \int_2^4 \left(\frac{9}{40}x - \frac{3}{80}x^2\right)\mathrm{d}x$ or alternatively computed $P(X > 2) = \int_2^4 \left(\frac{9}{40}x - \frac{3}{80}x^2\right)\mathrm{d}x$. Common errors included misinterpreting the expected fare calculation, often writing it as $E(Y) = 5X$, $E(Y) = 0.5X$, or $E(Y) = 3 + 0.5(2.4)$.

Answers: (*a*) $h = \frac{3}{24m - 64}$; (*b*)(i) $m = 6$, $h = \frac{3}{80}$; (ii) 15 ; (iii) $\frac{13}{20}$

Question 8

Many candidates opted for this question rather than Question 7, and most were able to correctly determine the unbiased estimates of the population mean and variance for tire lifespan. However, some used incorrect symbols like μ and σ^2 instead of the proper estimators $\hat{\mu}$ and $\hat{\sigma}^2$, while others simply referred to the estimates in words, "estimate of population mean and variance". Their understanding of units was generally sound-some expressed answers in thousands of kilometers, while others converted them to kilometers by multiplying by 1000. A frequent error was miscalculating the variance by using 2.5510×1000 instead of the correct 2.5510×1000^2.

Most candidates correctly identified $Z_{\frac{\alpha}{2}}$ as 2.170 or 2.171 and used the unbiased estimates from part (*a*) to calculate the standard error as $\sqrt{\frac{2.5510}{50}}$, which they then applied in the confidence interval formula. However, some mistakenly used the wrong standard deviation, $\sqrt{\frac{2.52}{50}}$, leading to inaccurate intervals.

In interpreting the confidence interval, many failed to include the phrase "97% confidence," instead referring to it as a "97% confidence interval," which weakened the clarity of their conclusions. Only a small number of candidates approached the hypothesis test methodically, beginning with correct hypotheses, identifying the critical value, calculating the test statistic as $\frac{4\ 8.5 - 48.0}{\sqrt{\frac{2.5510}{50}}}$, and comparing it to the critical value before making a decision about rejecting H_0. Some gave incorrect alternative hypotheses like H_1: $\mu < 48.0$ or $\mu \neq 48.0$, which led to wrong critical values. Many also repeated the earlier mistake of using the standard deviation instead of the standard error in their calculations.

Finally, most candidates provided a valid justification for the statistical method used, commonly citing the "large sample size", "sample size is 50", "sample size more than 30" or "sample size sufficiently large enough" as their reasons.

Answers: (*a*) $\hat{\mu} = 48.5$, $\hat{\sigma}^2 = 2.5510$; (*b*) (48.010, 48.990)

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