Mathematics T (954)

OVERALL PERFORMANCE

The number of candidates for this subject was 8109. The percentage of the candidates who obtained a full pass was 76.45%, with a decrease of 0.52% when compared to the results of the previous year.

The achievement of the candidates according to grades is as follows.

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	7.89	6.80	8.91	11.09	10.20	11.34	9.70	3.01	3.98	3.53	23.55

RESPONSES OF CANDIDATES

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General comments

The candidates' performances vary. Overall, the candidates' performance was average. For the above average candidates, generally they were able to plan and write their answers well. The good candidates were able to present and organized systematically written answers. In general, a majority of candidates did not put forward well-written answers questions which involve reasoning, deduction, functions and sketching of curves. Furthermore, candidates depended too much on the use of calculators, and as a consequence, the answers they give were not exact values. Candidates just stated the answers without showing essential steps to convince as to how the answers came about.

Comments on individual questions

Question 1

A very few candidates obtained full marks. A majority understood the needs of the question and was able to apply the laws of logarithm and solve the equation by the removal of ln. Since the problem involve ln, obtaining a precise answer was crucial to ensure the equation was fulfilled. However, most of the candidates changed e to 2.718, and gave the answer in decimal, which was inaccurate. Another common mistake was that quite a number of candidates did not verify their answer. As a result, they gave two answers but since $x > 0$, there was only one correct answer.

Answer: $x = \sqrt{(1 + e)} - 1$

Question 2

Most of the candidates were able to write $\frac{r^2 + r - 1}{r^2 + r}$ as $1 + \frac{1}{1 + r} - \frac{1}{r}$ using partial fraction. However, quite a number of candidates did not write the Σ notation and quite a number missed out on the 1. A majority understood the method of differences, whilst some were confused and used the series formula given in the question paper.

Question 3

Candidates were able to express the integral in terms of u but failed to apply integration by parts to solve the problem. Beside that weaker candidates were not able to do the substitution correctly.

Answer: $\frac{3}{2} - \frac{2}{e}$

Question 4

Most candidates were able to split the inequality into two cases and solved them correctly. However, quite a number failed to combine the answers to obtain the final solution set.

Quite a number of candidates also used the graphical method and obtained full marks for it. A common error in using the graphical method was not labeling the graph appropriately. A part from that, some candidates did not observe the symmetry of the graph $|x + 1|$.

Answer: $(-\infty, 2)$

Question 5

For candidates who understood implicit differentiation, they were able to differentiate implicitly to obtain the first derivative but quite a number failed to obtain and prove the second part. There were also quite a number of candidates who still failed to understand implicit differentiation and instead wrote $y\frac{\sin x}{\sqrt{x}}$ and differentiate explicitly.

A few of the candidates squared to get rid of the $\sqrt{\ }$ sign but could not proceed after second differentiation.

Answer: $k = \frac{2}{3}$

Question 6

Some candidates were able to answer the question systematically well. Quite a number were able to apply the distance formula, though some made mistakes in getting the wrong equation of line PQ. Beside that, while others made mistakes in substituting the correct values in the formulae. The question was intended to guide the candidates to determine the area of triangle PQR using the distance obtained in earlier part but most candidates calculated the area using other formulae which is also acceptable.

Common mistakes included not writing R in the coordinate form, failure to substitute correct values of $ax + by + c = 0$ into the distance formula, failure to substitute $m = 1$ when required and not simplifying the distance and area obtained.

Answers: (a) $R = \left(\frac{(b - a)m}{m^2 + 1}, \frac{m^2b + a}{m^2 + 1}\right)$;

(b) $\frac{b^2 - a^2}{2\sqrt{a^2 + b^2}}$;

(c) $\frac{1}{4}(b^2 - a^2)$

Question 7

Nearly all the candidates were able to answer the question in part (*a*) except for those who were weak in calculation. Most of the candidates were able to find $|z|$ but a majority failed to determine the arg z correctly.

Common mistakes included the representation of z and its conjugate in two separate diagrams, not labeling z and z^*, and just sketching the line without indicating the exact location of z and z^*.

Answers: (*a*) $z = -\sqrt{3} - \mathrm{i}$; (*b*) $-\frac{5\pi}{6}$

Question 8

A majority of them could differentiate e^{x^2} but failed to relate integration with differentiation. Many candidates failed to appropriately set the correct u and dv before performing integration by parts so that the result from the first part can be used. One possible choice is letting $\int x^3 e^{x^2} \mathrm{d}x = \frac{1}{2}\int x^2(2xe^{x^2})\mathrm{d}x$. Many of them just use normal integration by parts.

Answers: $2xe^{x^2}$, $a = 3$, $b = 2$, $c = 4$

Question 9

Almost all candidates were able to find $\mathrm{f} \circ \mathrm{f}$ but failed to deduce f^{-1}. Instead, they found the solution by using the conventional way.

Answers: (*a*) x; f^{-1}: $x \mapsto \frac{x}{2x-1}$, $x \neq \frac{1}{2}$; (*b*) $a = 1$, $b = 0$, $c = -1$; (*c*) $\mathrm{h}(x) = (\mathrm{f} \circ \mathrm{p})(x)$

Question 10

A majority of the candidates answered question (*a*) well and obtained full marks. For those that did not get full marks, their errors were usually in the calculation of $|\mathbf{A}|$. There were weak students who got confused and mixed up in locating the adjoint, determinant and cofactor.

Many candidates just used the calculators to find $\mathbf{A}^{-1}$, and as a result, they did not get any marks since they did not use the adjoint to obtain the answer.

A majority of the candidates were able to find **B**. The common error in this question was that candidates post-multiplied instead of pre-multiplying.

Answers: (*a*) $|\mathbf{A}| = 2$, $\mathrm{adj}\ \mathbf{A} = \begin{pmatrix} -2 & 3 & 0 \\ 2 & 0 & 4 \\ 0 & 2 & 2 \end{pmatrix}$ $\mathbf{A}^{-1} = \begin{pmatrix} -1 & \frac{3}{2} & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix}$; (*b*) $\mathbf{B} = \begin{pmatrix} -8 & 27 & 0 \\ 9 & 2 & 18 \\ 0 & 18 & 10 \end{pmatrix}$

Question 11

Almost all the candidates answered well using the remainder and factor theorem to obtain the values for a and b, and were able to factorise $\mathrm{p}(x)$ completely. However, in part (*b*), a majority of the candidates did not state or indicate that $x \neq 3$.

In part (*c*), a common error amongst the answers given was that they wrote $2x^2 + x - 1 = x^2 + \frac{x}{2} - \frac{1}{2}$.

Some candidates were also confused between the minimum value and minimum point, and as a result, gave their answers in coordinate form.

Answers: (a) $a = 2,\ b = -5;\ p(x) = (x + 1)(2x - 1)(x - 3);$

(b) $\left\{x : x \leqslant -1,\ x \geqslant \frac{1}{2},\ x \neq 3\right\}$; (c) min value is $-\frac{9}{8}$ when $x = -\frac{1}{4}$

Question 12

This is the question that very few candidates were able to answer in full.

There were some candidates who were able to state the asymptotes but a majority did not actually give the exact answer. They usually gave more than the required which indicated that the candidates did not understand the concept of asymptotes.

Most of candidates could not differentiate ln $2x$ correctly as required in part (b).

They used the calculator to solve the equation $f'(x) = 0$ for x which does not give the exact answer in terms of e. In general, candidates knew how to find and determine the type of stationary points, but due to careless differentiation in earlier parts, marks were lost.

Almost all candidates did not understand the concept of concavity and inflexion points as required in part (c). This particular part of the question was poorly answered. The graph was also not perfectly well-sketched.

Answers: (a) $x = 0,\ y = 0,$

(b) $\left(\frac{1}{2}e^{\frac{1}{2}}, \frac{2}{e}\right)$ is a maximum point.

(c) (i) concave up when $(1.15, \infty)$

(ii) concave down when $(0, 1.15)$

The inflexion point $= \left(\frac{1}{2}e^{\frac{5}{6}}, \frac{10}{3}e^{-\frac{5}{3}}\right)$ or $(1.15, 0.630)$

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General comments

In general, the questions are precise, well-presented and were of good quality. The paper covers the whole spectrum of topics relevant to the Form Six Syllabus. There were also good questions not of the routine text book types for which the candidates required thorough understanding of the subject matter. The distribution of marks was fair and reasonable.

Comments on Specific Questions

Question 1

There were good answers from good candidates. However, the weaker candidates failed to recognize the appropriate 'starting points', (factor formula) and went on expanding the terms to non-manageable expression.

Question 2

This is a good question with a clearly-stated task. The strong candidates answered well. Candidates that were weak in vector concepts were not clear about the nature of the scalar product and made glaring mistakes such as $c \bullet b = c \bullet d \Rightarrow b = d$.

Question 3

The first part was well-answered by many candidates. However, there were candidates who misunderstood 'KJ extended to T' resulting in the wrong orientation of the diagram. Many candidates failed to use the side ratios to prove the second part. Instead, they used the Pythagoras theorem and ended up in mass.

Question 4

Generally, the first part of the question was well-answered. Candidates were able to show $\frac{du}{dx} = \frac{2u}{x(u-1)}$ but failed to give the final answer in any of the equivalent form after successfully solving the differential equation. Another weakness observed among the candidates was the inability to manipulate log function to simplify the answer.

Answer: $y = 4x + x\ln xy$ or $xy = e^{\frac{y}{x}-4}$

Question 5

This question was poorly answered. Many candidates could not visualize simple vector additions which lead to the solution. A majority could not achieve the full mark because they could not prove in terms of vectors that the medians were concurrent.

Question 6

This is not a difficult question and most of the candidates were able to answer part (*b*) and (*c*), many could deduce the maximum and minimum values but failed to associate them to the corresponding *x*-values. The sketching of the curve was poorly done and many did not appreciate and use points already obtained in the earlier parts. Beside that, there were some candidates who were unable to correctly sketch the straight line.

Answers: (*a*) $r = 2$ and $\alpha = \frac{\pi}{3}$

(*b*) minimum value $= \frac{2\pi}{3}$ and maximum value $= \frac{5\pi}{3}$

(*d*) values of $x = \frac{\pi}{3}, \pi$

Question 7

Many candidates could answer the question as they applied straight usage of the concept of selection without replacement or ${}^{n}C_{r}$. However, there were candidates who mistook ${}^{40}C_{2}$ as ${}^{40}C_{1} \times {}^{39}C_{1}$.

Answers: (*a*) $\frac{17}{33}$, (b) $\frac{92}{225}$

Question 8

This question was well attempted. Some candidates did not use the Poisson distribution, as the approximate distribution, instead they used binomial or even Normal distribution. Apart from that many were unable to apply linear combination of random variables in Poisson distribution.

Answers: (*a*)(i) 0.36788, (ii) 0.01899, (*b*) 0.27067

Question 9

Many candidates failed to use the definition to find E(2X – 1) in which the integral would be easier to perform. They tried to obtain E(X) which was more difficult to integrate and were prone to mistakes. Some even treated the variables as discrete. Besides that many failed to obtain Var(X) as they could not find E(X).

Answers: (*a*) $\mathrm{E}(X) = \frac{8}{3}$,

(*b*) $\frac{61}{45}$

Question 10

In general, the performance on this question was satisfactory. Most of the candidates were able to use the correct formulae to find mode, median and the mean. However, there were some common mistakes such as making continuity correction on the already given boundaries of the required classes, the thus was lower boundary for median L = 1600.5. Candidates also lost marks for not giving the final answer to the nearest hour.

Answers: (*a*) Median = 1627, mode = 1621,

(b) $k = 1875$

Question 11

Most of the candidates understood what was required of the question, and those who used a table for the outcomes of the experiment were able to draw out the required distribution. A common mistake among the candidates was the wrong perception of word 'difference', thus, giving wrong values of the variables. Apart from that the candidates were weak in transforming $|X - E(X)| \leqslant 1$ into non-modulus expression which also contributed to the loss of marks.

Answers: (*b*) $E(X) = \frac{5}{4}$,

(c) $\frac{5}{8}$

Question 12

Generally well-answered especially in part (*a*). Again many candidates did not give the answers in percentage as required of the question. Part (*b*) was a disaster for many because they failed to realise that it was on conditional probability. Part (*c*) was well-answered but part (*d*) lost many marks for failure to acquire the correct mean and variance for $X_1 + X_2$.

Answers: (*a*) small = 26.60, medium = 62.84%, large = 10.56%,

(*b*) 0.77784,

(c) 0.07075