Mathematics T (954)

OVERALL PERFORMANCE

The number of candidates for this subject was 7967. The percentage of the candidates who obtained a full pass was 68.09%, a increase of 2.16% when compared to the previous year.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	9.15	6.82	9.29	10.95	10.02	10.71	11.15	3.02	4.23	3.16	21.50

RESPONSES OF CANDIDATES

PAPER 954/1

General comments

Generally, there were mixed performances from the candidates. The good candidates did not face much difficulty in answering and presenting their answers. The answers given were properly planned and well-organized. These candidates showed full understanding of the questions and concepts and also possessed the manipulative skills required of them.

The moderate candidates seem to understand the questions. However, they tend to make careless mistakes. They were not able to answer well the more difficult questions.

In contrast, the weaker candidates were weak in many aspects, such as in understanding the question, not knowing the concept, not knowing or remembering the formulae, and their answers were not well presented. The candidates were very weak in answering questions which involved reasoning, deduction and sketching of two curves.

Generally, many candidates were poor in presenting mathematical arguments logically. An example is seen in question 9, where they were asked to show that the function is decreasing. As for integration, they knew integration by parts but could not proceed integrating tan x. Candidates were also very weak in functions. They didn't understand the meaning of composite functions. Generally, candidates depended too much on the calculator, and thus, sometimes they were not able to think correctly of the answers. For example, given a polynomial, the candidates used the calculator first to obtain the roots or factors before actually factorizing the polynomial. Similarly, candidates also used the calculator to evaluate certain value of log such as $\log_3 y = \frac{1}{2} \Rightarrow y = 1.732$ instead of $\sqrt{3}$ which is the exact value of y.

Comments on individual questions

Question 1

This question requires candidates to use $T_n = S_n - S_{n-1}$ to determine the nth term. Then, using this result, deduce the type of progression by showing that $T_n - T_{n-1} = 6$ which is a constant. It is a straight forward question, but quite a number of candidates answered differently. In some cases, candidates first find the values of S_1, S_2 and S_3 followed by T_1, T_2 and T_3. Candidates then deduced that the common difference is 6, and thus, an arithmetic progression without first determining the nth term T_n. Quite a number of the candidates used $T_n = a + (n - 1)d$ to prove that it is an arithmetic progression without finding the nth term.

Answer: $T_n = 6 - 3n$

Question 2

In answering this question, candidates were expected to take log (ln) on both sides and to perform implicit differentiation. However, candidates were not familiar with variable x as the index (raised to power). Quite a number of candidates were confused between $(2x)^{2x}$ with a^x. Some candidates differentiated as $\frac{dy}{dx} = (2x)(2x)^{2x-1}$. Some candidates differentiated ln $(2x)$ as $\frac{1}{2x}$ and forgot to multiply it by 2.

Answer: $\frac{dy}{dx} = 2(2x)^{2x}[1 + \ln(2x)]$

Question 3

In this question, candidates were required to show $\frac{d}{dx}(\tan x) = \sec^2 x$. However, quite a number of candidates failed to express $\tan x = \frac{\sin x}{\cos x}$ and then differentiated it using the quotient rule. But the candidates did manage to use the result $\frac{d}{dx}(\tan x) = \sec^2 x$ in the next part of the question. Many candidates were able to apply the result in the integration by part of $\int_0^{\frac{\pi}{3}} x \sec^2 x \, dx$. There were also a significant number of candidates who did not know how to integrate tan x. Some evaluated it as $\int \tan x \, dx = \ln \cos x$ and forgot the negative sign. There were also candidates who did not know that $\tan \frac{\pi}{3} = \sqrt{3}$.

Answer: $\frac{d}{dx}(\tan x) = \sec^2 x$

Question 4

Majority of the candidates were able to use the factor theorem to solve for a and b. But quite a number of them factorized p(x) wrongly. They wrote $2 - x - x^2 = (x + 2)(x - 1)$. Although, they could obtain the values for a and b, the factorization for $p(x) = (x + 2)(x - 1)(2x + 1)$ led them to the wrong answer. Some candidates did not give their answers in the set form.

Answers: $a = -2$, $b = 5$; $\{x: -2 < x < \frac{1}{2}, x > 1\}$

Question 5

This is a straight forward question which most candidates were able to answer. They knew how to find the inverse of a matrix. Those who could not get the correct answer are mostly due to their carelessness in multiplication of matrices.

A number of candidates were confused with the cofactor, adjoint and transpose of a matrix. Some candidates did not express the matrices $\mathbf{A}^2$ and $\mathbf{A}^3$ in full and complete forms. Instead, they just evaluated only a few of the elements. This caused them to lose marks as marks are allotted for complete matrices.

Answer: $x = 2$

Question 6

This question was not answered well by the candidates. Many of them could not understand the question, and did not know what a composite function was. Some candidates expressed the composite function of f and g as the product of the functions.

Some of candidates did not have the skill to express x in terms of y or vice versa when finding $g(x)$. For candidates that were able to determine $g(x)$, some were not able to obtain the domain.

Answers: (a) $gx = \sqrt{\ln x - 2}$; $\{x : x \geqslant e^2\}$ (b) e^3

Question 7

Most candidates were able to answer this question well. Common mistakes made by the candidates were using the wrong formula/properties of logarithm and writing $(\log_3 x)(\log_3 y) = \log_3(x + y)$, $(\log_3 x)(\log_3 y) = \log_3 (x) + \log_3 (y)$, and $(\log_3 x)(\log_3 y) = \log_3 (xy)$. Some candidates gave their answers in decimal form instead of the exact value.

Answers: $x = 9$, $y = \sqrt{3}$; $x = \dfrac{1}{\sqrt{3}}$, $y = \dfrac{1}{9}$

Question 8

Most candidates were able to express the expression in partial fraction form. However, when showing sum to the nth terms, a majority of them did not show the second last term. As for the last part, when finding the sum to infinity, many candidates tended to write $\frac{1}{\infty}$ for $\lim\limits_{n\to\infty} \frac{1}{n}$. Some candidates did not write appropriately or did not know that $S_\infty = \lim\limits_{n\to\infty} S_n$. They also did not multiply with $\frac{1}{3}$ to get the final answer.

Answers: $\dfrac{3}{(3r - 1)(3r + 2)} = \dfrac{1}{3r - 1} - \dfrac{1}{3r + 2}$, $\dfrac{1}{6}$

Question 9

In part (a), some candidates expressed $f(x) = \frac{u}{v} = uv^{-1}$. So, instead of the using quotient rule to differentiate, they used the product rule. Generally, most candidates were able to differentiate using either the quotient or product rule.

In part (b), there were quite a number of the candidates that did not know how to show that $f'(x) < 0$. There were also candidates that did not know the concept of a decreasing function. Some were not aware of the relationship between negative gradient and decreasing function. Some candidates did not show that $x^2 + x + 1 > 0$, instead they just substituted the value. Basically, many candidates did not argue that since $1 + x + x^2 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} > 0$, $(1 + x^2)^{\frac{3}{2}} > 0$ and $e^{-x} > 0$, thus, $f'(x) < 0$, which implies that f is a decreasing function. The candidates failed in completing the square and to conclude that f is a decreasing function. However, many were able to give a sketch of the graph.

Question 10

In part (a), quite a number of candidates were not able to find the inverse of f since they did not realize that they had to complete the square in order for them to express x in terms of y for $y = x^2 - x$. However, some of these candidates do realize that the domain of f^{-1} is the same as the range of f. So, they were able to state the domain of $x^2 - x$.

In part (*b*), quite a number of candidates did not realize that the intersection point of f and its inverse can also be found by finding the intersection of $x^2 - x$ and x since f and f^{-1} intersects on the line $y = x$.

In part (*c*), the candidates were able to sketch both graphs of f and its inverse since they knew the reflection property of f about the line $y = x$ gives the graph of f^{-1}.

Answers: (*a*) $f^{-1}(x) = \frac{1}{2} + \sqrt{x + \frac{1}{4}}$; Domain $f^{-1} = \left\{x : x \geq -\frac{1}{4}\right\}$; (*b*) (2, 2);

Question 11

This is a relatively easy question. Most candidates were able to answer part (*a*). As for part (*b*), a majority of the candidates were able to determine the equation of tangents by finding the derivative using implicit differentiation. For the weaker candidates, they failed to understand the question and went to find a line passing through A and B instead. Many candidates were also able to find the shortest distance using the distance formula since the equation of the line AB is given.

Answers: (*a*) $A(-\frac{1}{2}, 2)$, $B(1, -1)$; (*b*) $C\left(\frac{5}{2}, 5\right)$; (*c*) $d = \frac{9}{\sqrt{5}} = \frac{9\sqrt{5}}{5} = 4.02$

Question 12

In part (*a*), majority of the candidates understood the concept of rationalizing and writing z^2 in terms of its real and imaginary parts. However, many failed to understand that the imaginary part is a real number, and instead most of them wrote $-\frac{4}{25}i$ for the imaginary part. However, the candidates were still able to solve the equation to determine z_1 and z_2.

The aim of part (*b*) of this question is to establish the relationship between the roots of a real polynomial. Generally, in any polynomial, the roots need not be conjugates of each other. This property only holds for real polynomial. Candidates that answered the question in the sequence given and solved for a and b could easily find the second root and deduced the answer. However, there were some candidates that assumed the conjugate relationship before actually verifying it and used it to solve for a and b.

Answers: (*a*) $z^2 = \frac{3}{25} - \frac{4}{25}i$, Real part $= \frac{3}{25}$, Imaginary part $= -\frac{4}{25}$, $z_1 = \frac{1}{5}(2 - i)$, $z_2 = \frac{1}{5}(-2 + i)$;

(*b*) $a = 4$, $b = 1$ or $a = -4$, $b = 1$;

(*c*) $z_3 = \frac{2}{5} + \frac{1}{5}i$ or $z_3 = -\frac{2}{5} - \frac{1}{5}i$; z_1 and z_3 conjugate of each other

PAPER 2 (954/2)

General comments

Generally, the quality of answers given by candidates varies. They showed a wide range of Mathematical abilities. Good candidates gave well planned answers and systematic steps in their presentations and working. On the other hand, weaker candidates showed a lack of understanding of the requirements of the questions. They even performed poorly in planning, argument and reasoning skills as in deductive geometry.

Comments on individual questions

Question 1

Many candidates were familiar with the product rule in differentiation, but some candidates were confused and let $y = ye^{-x}$ before applying the rule. Some candidates failed to observe that the next part requires them to use the answer from the first part. These candidates did not use the anti-derivative concept to solve the problem but used the integration by parts instead.

Many candidates used wrong substitutions because they were confused about the 'y' in $\frac{dy}{dx}$ and the 'y' in the given differential equation, and hence, committing the following errors:

(i) $\frac{dy}{dx} = e^{-x}\left(\frac{dy}{dx} - y\right)$

$e^{-x}\left(\frac{dy}{dx} - y\right) - y = e^{x}\cos x$

(ii) $e^{-x}\left(\frac{dy}{dx} - y\right) = 0$

$0 = \cos x$

Generally, those who substituted correctly could get the particular solution of the differential equation. Weaker candidates did not perform well in this question as it requires a strong understanding of the concept and relationship between differentiation and integration.

Answers: $e^{-x}\frac{dy}{dx} - ye^{-x}$, $ye^{-x} = \sin x + 1$

Question 2

There were a number of candidates who had problems with mathematical terms such as "similar and congruent triangles".

In part (*a*), many candidates could show that ΔBFC is similar to ΔBCA but many of them assumed that $\angle ACF = 90°$ and/or $\angle CBF = \angle ABC = 90°$.

In part (*b*), candidates were expected to use deductive geometry to show with reasons, but some weak candidates could not use proper reasoning.

Common errors found were as follows:

$\angle BCF = \angle CAB$ (alternate angle),
$\angle BCF = \angle CAB$ (exterior angle equals to opposite interior angle),
ΔBFC and ΔBCA are similar (SSS), (SAS).

Question 3

Many candidates could plot the graph. The graph appears like a parabola with incomplete labels. Some candidates did not use the double angle formula to find θ. This is because they could not relate the inequality to the sketch of the graph. Those who could solve, however, did not write the final answer in the set notation. Some of the common errors found involved sketching the graph of $y = \cos\theta$ for $0 \leqslant \theta \leqslant \pi$ and sketching the graph of $y = \cos 2\theta$ for $0 \leqslant \theta \leqslant 2\pi$.

Answer: $\left[\frac{\pi}{12}, \frac{11\pi}{12}\right]$

Question 4

This question was poorly performed by the candidates. Most of them could find $\overrightarrow{PS}$, but could not interpret 'the position vector of any point on *PS*'. This led to the problem of answering part (*b*) and part (*c*). Many of the candidates used the same symbol, λ, to express the position of the vector of any point on *QT*. They did not understand the question well enough to solve the problem. Failure to see *N* as 'any point on *PS* and *QT*' made the candidates unable to obtain the remaining 7 marks.

Answers: (*a*) $\frac{1}{2}(\mathbf{q} + \mathbf{r}) - \mathbf{p}$; (*b*) $(1 - \mu)\mathbf{q} + \frac{\mu}{2}(\mathbf{r} + \mathbf{p})$; (*c*) $\frac{1}{3}(\mathbf{p} + \mathbf{q} + \mathbf{r})$

Question 5

Most candidates could answer part (*a*), but there are some who assumed that $\angle ABC = 120°$ in order to obtain $\Delta EBD = 60°$. Most candidates could not answer part (*b*) well because they used the wrong formula to find the area. Those who were able to solve part (*b*), mostly did not give the answer in the form of ratio, but in the form of decimal instead such as $\frac{45}{79}$.

Answers: (*a*) $\angle EBD = 60°$; (*b*) 45:79

Question 6

This is the most attempted and well performed question. However, many candidates made the mistake by assuming an arbitrary constant $\mathrm{A} = \frac{1}{\mathrm{A}}$, in order to show $u = \frac{x}{\mathrm{A} - x}$. Many candidates could not perform algebraic operations to reach $u = \frac{x}{\mathrm{A} - x}$. Some candidates did not simplify the final answer until $y = \frac{2x^2}{3 - 2x}$. The answer was left as $y = \frac{x^2}{\frac{3}{2} - x}$.

Quite a number of candidates made the mistake in performing the antilog of an equation such as:

$\ln u - \ln(1 + u) = \ln x + \ln c \longrightarrow \frac{u}{1 + u} = c + x$ and $\ln u - \ln(1 + u) = \ln x + c \longrightarrow \frac{u}{1 + u} = cx$

However, good candidates could solve the partial fraction correctly with only a few short steps.

Answer: $y = \frac{2x^2}{3 - 2x}$

Question 7

Majority of the candidates could not answer this question as they failed to understand the requirement of the question or it seems confusing to them. Many of them could not grasp the idea of choosing 3 as in ${}^3C_3 \times {}^6C_0$ or ${}^3C_2 \times {}^6C_1$. Some of them assumed it as a permutation problem and some were not sure whether to use ${}^nC_r$ or ${}^nP_r$. Quite a number of the candidates calculated the probability using binomial. However, the few candidates who could answer the question mostly scored full marks.

Answers: (*a*) $\frac{19}{84}$; (*b*) $\frac{5}{21}$

Question 8

Many candidates could solve part (*a*) because of the simple and direct task of standardizing a normal distribution. However, many failed to interpret part (*b*). They were confused and understood the question as $P(X > t) = 0.05$ instead of $P(X \leqslant t) = 0.05$.

Answers: (*a*) 0.159; (*b*) 134

Question 9

One of the most attempted question, but not understood well by the candidates. Some candidates mixed up the coding method with the summation method to find the mean and standard deviation. Some candidates used wrong properties of $\sum$ to find $\sum x$ and $\sum x^2$ while some used wrong formula to find the standard deviation. Very few candidates could obtain $\sum x = 76000$.

Answers: 304; 2

Question 10

A very well answered question, but some candidates failed to use the formula correctly to find the median and quartiles.

For part (*a*), some of them use the correct formula but substitute the wrong midpoint or lower boundaries and class width of respective classes for the median, first quartile and third quartile, and hence, they could not get all the three values correctly.

Very few candidates could answer part (*c*) correctly because they failed to use the information in part (*b*) to calculate the values of $Q_2 - Q_1$ and $Q_3 - Q_2$, in order to support the comment made on the shape of the distribution.

Answers: (*a*) 6.23; (*b*) 6.25, 6.08, 6.39; (*c*) Negatively skewed

Question 11

Many candidates could not interpret the distribution function, and hence, gave all sorts of wrong answers. They were supposed to use discrete distribution but they used continuous distribution instead, and hence, they used integration to calculate the probability, mean and variance. Some candidates used the wrong formula $P(X \geqslant 9) = 1 - P(X \leqslant 9)$ to calculate part (*b*).

For part (*c*), those who got part (*a*) correct, generally knew how to find $E(\sqrt{X})$ and $E(\sqrt{X})^2 = E(X)$ as well as used the formula $\mathrm{Var}(X) = E(X) - [E(\sqrt{X})]^2$ to calculate the value of variance.

Answers: (*a*)

x	1	4	9	16	25
$P(X = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

(*b*) $\frac{3}{5}$; (*c*) 3, 2

Question 12

The performance of the candidates on this question is not good. However, some candidates performed well in part (*a*) (i) and part (*c*). Majority of the candidates failed to answer part (*a*)(ii) correctly because they were weak in conditional probability.

While for part (*b*), most candidates failed to use the concept of
P(two independent events) = P(Event) × P(Event). Most of them could only obtain $P(N') = 0.1252$.

For part (*c*), most candidates knew how to use the binomial distribution to determine the probability.

Answers: (*a*) (i) 0.8748; (*b*) 1.57%; (*c*) 0.120