

# MATHEMATICS (T) (954/1)

## OVERALL PERFORMANCE

The number of candidates for this subject was 4 016. The percentage of candidates who obtained a full pass was 49.46%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	6.80	3.46	3.44	5.43	9.24	8.69	12.40	5.88	6.62	6.20	31.85

## CANDIDATES' RESPONSES

### PAPER 954/1

#### *General comments*

Overall, the presentation of the candidates' solutions revealed mixed performances of their mathematical ability. There was a wide range of mathematical abilities amongst the candidates. Generally, most candidates were able to answer questions which required direct applications of mathematical formulae but weak in answering questions involving understanding concepts. Almost all candidates attempted all seven questions and in quite a significant number of answers, the workings were clearly shown. Straight forward questions that were familiar to students such as the first part of Q2, Q3 and Q4 were answered very well as these questions only required them to use standard mathematical method and basic concepts. Achievements in part 1 of Q1, Q5 and Q6 were good. The performance in Q7 and Q8 were slightly poor. A few number of candidates did not show sufficient steps or made clear reasons(s) that led to their answers. In another aspect, there were candidates that did not provide exact answers as required; such as Q1 and Q6 where answers were given in decimal form and angles were not given in terms of  $\pi$ . Candidates' answers show that they were still having problems when facing questions which involved "hence". Quite a number of candidates did not use the previous result attained to solve the following part of the respective questions.

For good candidates, their answers were well organised and presented. The candidates produced precise and concise answers. High achiever candidates answered systematically and strategised the steps presented, showing their full understanding of the questions and concepts. Their performance in Q1, Q2, Q3, Q4, Q5 and Q6 were excellent with almost perfect score in these questions. Excellent candidates were able to present exemplary well with essential workings as these questions only required them to use standard mathematical method and basic concepts.

Moderate candidates attempted all seven questions but a majority gave partially completed solutions. In some moderate students, they were able to answer most of the first part of the questions and usually struggle with the latter part of the question which required enhanced knowledge and thinking skills; such as in Q1, Q2, Q4, Q7 and Q8. They presented their answers tremendously well for the questions they were familiar with. Nonetheless, they tend to make careless mistakes and having difficulty in answering more challenging questions such as Q7 or Q8. However, some of these moderate candidates lose marks in the second part of Q1, Q2 and Q4.

For the weak students, they lack understanding and knowledge in the fundamentals in mathematical concepts. Some of these weak candidates did not even know how to apply formulae already given such as

in Q1 and Q5. They also performed badly in most of the questions especially those requiring analytical formulae and thinking skills. As a result, they lacked planning in how to answer and their answers were mostly incorrect. These candidates did not know how, why and when to use the concepts. They wrote messy answers, using wrong formula and wrong mathematical principles. Some solutions given were meaningless. Almost all of these weak candidates were not able to solve Q1(b), Q2(b), Q5, Q6 and Q7 or Q8. Poor attempts were seen as the solutions provided by them were untidy and pointless. As a whole, most of the weaker candidates were not able to organise, having wrong concept, not able to plan and write their solutions systematically.

As a whole, the performance of candidates of semester 1, 2022 were satisfactory. Candidates' use of language were good and clear. Almost all scripts were answered in English. Not to mention, some candidates still used two columns in presenting their working, split a page to two, squeezing two questions to one side making a page with solutions of three to four questions which make it hard for the examiners to write marks at the appropriate place. Some candidates still produced small and unorganised set of answers. Consequently, their scripts were totally packed and difficult to mark. A number of candidates did not indicate the question or the part of the question which they were attempting. Marking were difficult with illegible handwriting.

### Comments on individual questions

#### Question 1

Almost all candidates attempted this question and mostly managed to use the correct compound angle formula namely;  $\cos(A - B) = \cos A \cos B + \sin A \sin B$ . This formula was provided in the formula list and

candidates correctly expressed  $\cos\left(\frac{\rho}{2} - q\right) = \cos\frac{\rho}{2} \cos q + \sin\frac{\rho}{2} \sin q$  to obtain the first two marks. Most

candidates were not able to use "hence" in order to solve  $\cos 3x = \sin\frac{\rho}{14}$ . Candidates missed out the

relation  $\cos\left(\frac{\rho}{2} - \frac{\rho}{14}\right) = \sin\frac{\rho}{14}$ . Many candidates used  $\cos^{-1} \sin\frac{\rho}{14}$  to find  $3x$  and thus expressed their  $x$

values obtained in decimals. Most candidates successfully used the correct formula to show the relation and quite a number managed to solve values of  $x$ . Sketching the graph was not a problem for most of the candidates and candidates who got the correct values of  $x$  in the first part, mostly would obtain the last one mark for the inequality. Quite a large number of candidates did not use the relation proven to solve the

equation given and hence, not answering the question. Few candidates managed to get  $3x = \frac{3\rho}{7}$ , but they

were not able to find another two corresponding angles correctly or changed their  $x$  values obtained into decimals. Even though candidates succeeded in sketching the graph but not obtaining the correct values of  $x$  in the first part affected the final answer.

$$\text{Answers: } x = \frac{\rho}{7}, \frac{11\rho}{21}, \frac{17\rho}{21}; \quad \begin{cases} x: 0 < x < \frac{\rho}{7} \\ x: \frac{11\rho}{21} < x < \frac{17\rho}{21} \end{cases}$$

#### Question 2

Only a handful of candidates scored full marks for this question. Most candidates successfully got the constants in the partial fraction in part (a) but many lose marks in part (b). Most candidates excellently got the correct values for  $P$  and  $Q$ . Seeing the relation of the series given with the general terms and connecting them with the partial fraction, they splendidly found the sum of the first  $n$  terms of the series using the method of difference. Some careless mistakes occurred when solving for the constants. Most candidates did not make use of the "hence" mentioned. Mostly did not write the series in terms of the given general terms (marks deducted due to "no essential working"). Quite a number of candidates wrote only two first terms and one last term while using the method of difference to attain the sum. Again, marks deducted for "not well presented working".

$$\text{Answers: } P = \frac{1}{3}, Q = -\frac{1}{3}; S_n = \sum_{r=1}^n \frac{1}{(3r+1)(3r+4)} \dots = \frac{1}{3} \left( \frac{1}{4} - \frac{1}{(3n+4)} \right) = \frac{n}{4(3n+4)}$$

### Question 3

Matrices was one of the favourite questions and usually was the best performed question. Most candidates had the ability to perform the elementary row operation (ERO) perfectly and got the correct inverse except for some calculation error by a few candidates. Almost all candidates attempted this question and mostly succeeded in obtaining the first six marks. Mostly wrote the augmented matrix correctly. In some of the candidates, the use of long and short arrows (writing the operations appropriately) caused them to lose a few marks for not presenting answers well. A few candidates wrote improper ERO instruction and left out the short and long arrows signs.

$$\begin{array}{cccc|c} x & 5 & 1 & -1 & 0 \\ y & 14 & 2 & -3 & 5 \\ z & -4 & -1 & 1 & 0 \end{array}; \quad x = 3, y = 5, z = -2$$

### Question 4

Good performance from a majority of the candidates for the first part of the question. The candidates mostly achieved complete and perfect answers. Almost 95% of the candidates were able to get the correct modulus and argument and finally, obtained the correct polar form. Quite a number of candidates were capable of using de Moivre's correctly to attain  $z^n$ . Some candidates used the trial and error method by replacing  $n = 2$  and  $n = 3$ . This would direct them to the answer whereby  $z^2 = \cos \frac{2\rho}{3} + i \sin \frac{2\rho}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$  and  $z^3 = \cos \rho + i \sin \rho = \cos \rho = -1$ . Quite a number of candidates expressed imaginary part as  $i \sin \frac{n\rho}{3}$  instead of  $\sin \frac{n\rho}{3}$ . A few non-attempts on this part most probably due to lacking the concept relating a complex number with a real number.

$$\text{Answers: } z = \cos \frac{\rho}{3} + i \sin \frac{\rho}{3}; n = 3$$

### Question 5

Performance of candidates in this question was moderate even though the formula was given in the question paper. Candidates excellently selected the correct formula for the parabola and managed to find the constant 'a' using the vertex and passing point given. Candidates successfully wrote correct standard form of equation of parabola. Quite a number of candidates picked the wrong formula and ended up getting incorrect equation of parabola. Few candidates did not make use of the information given to find the constant 'a' even though the correct formula had been selected. Quite a number of candidates attempted to find 'a' using  $a = -3 - (-1)$  due to wrong assumptions that  $(-3, 1)$  was the focus of the parabola.

$$\text{Answers: } (y + 3)^2 = -4(2)(x + 1)$$

### Question 6

This was a moderately performed question which was well attempted by majority of the candidates. They could either use  $\overline{AB} = \overline{DC}$  or  $\overline{AD} = \overline{BC}$  or the concept of mid-point of diagonals  $\overline{AC} = \overline{BD}$  to find the position vector of  $D$ . Most candidates were capable to get position vector of  $D$  using correct alternatives. Subsequently, proceed with any two vectors that connect point  $D$  using the formula of cosine to attain the correct angle,  $\angle ADB$ . Most candidates who successfully identified the appropriate vectors succeeded in computing the area of a parallelogram  $ABCD$ . Few candidates could not find the position vector of  $D$  because of the labelling of the parallelogram which was in an incorrect manner. Due to that, wrong position vector of  $D$  was obtained and led to the wrong angle and candidates who proceeded with the same  $D$  obtained, they calculated the area of the parallelogram  $ABCD$  incorrectly.

Answers: (a)  $6\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ ;  $\angle ADB = 43.29^\circ$ ; (b)  $\sqrt{150}$  or  $5\sqrt{6}$

### Question 7

Candidates who attempted this question with excellent understanding of function concept, managed to attain good marks. Most candidates managed to answer the first part of (a) correctly. Finding  $f(p)$  and relating both functions was not a problem to the candidates. Some candidates used the equal intermediate function of  $\frac{p^2 + 4}{p}$  to show the relation of the two functions. Almost all candidates could not deduce "f is not a one-to-one function" using the relation  $f(p) = \frac{p^2 + 4}{p}$ , which they had proven in the first part. Candidates concluded that  $k = 2$  from the given graph and instantly gave the range of  $f$  without justifying the minimum point in order to attain the smallest value of  $k$  such that  $f$  is a one-to-one function. Most candidates used dummy variable to express  $x$  to obtain the inverse of  $f$  but mostly could not proceed further. Few candidates managed to reach up to  $y = \frac{x}{2} \pm \frac{\sqrt{x^2 - 16}}{2}$ , but not able to choose the correct answer from the result obtained. Few candidates managed to proceed but did not provide the correct reason for the selection of the function for  $y$ . Almost all candidates were not successful in showing that  $y = f(x)$  lies above the line  $y = x$ . Some drew the graph of  $y = f(x)$  and  $y = f^{-1}(x)$ , and indicated no intersection between them and deduced the number of solution is 0.

Answers: (b)  $k = 2$ ;  $R_f = \{y : y \geq 4\}$  or  $[4, \infty)$ ; (c) (i)  $f^{-1}(x) = \frac{x}{2} + \frac{\sqrt{x^2 - 16}}{2}$ ,  $x \geq 4$ , (ii) No of solution = 0

### Question 8

Candidates who chose this question mostly got more marks as compared with candidates who chose Q7. Most candidates were able to find  $\overline{OC}$  and  $\overline{OD}$  correctly using  $\overline{OC} = \overline{OT} + \frac{1}{2}(\overline{OP} - \overline{OT})$  and  $\overline{OD} = \overline{OT} + \frac{1}{2}(\overline{OR} - \overline{OT})$ . Generally, candidates managed to get the correct relation of  $\overline{AC}$  and  $\overline{BD}$  to attain the vector equations of lines  $AC$  and  $BD$  in terms of  $q$ . Those who successfully attained the vector equations of lines  $AC$  and  $BD$  managed to get the correct direction vector to proceed with finding the position vector of  $U$ . By using the position vector  $U$  attained, then utilised the dot product of  $\overline{OU}$  and  $\overline{TQ}$ , and equated to zero, candidates successfully got the exact value of  $q$ . Candidates who were not able to relate

$\overrightarrow{OT} = q\mathbf{k}$  could not get the two vectors;  $\overrightarrow{OC}$  and  $\overrightarrow{OD}$ . Quite a number of candidates equated  $\overrightarrow{OT}$  to  $q$  which was conceptually wrong leading to the whole presentation of answer in a wrong manner. Most candidates were not able to answer this question mainly because of the incorrect  $\overrightarrow{OC}$  and  $\overrightarrow{OD}$  found previously.

Answers: (a)  $\overrightarrow{OC} = 2\mathbf{i} + 2\mathbf{j} + \frac{q}{2}\mathbf{k}$ ,  $\overrightarrow{OD} = -2\mathbf{i} - 2\mathbf{j} + \frac{q}{2}\mathbf{k}$ ,  $\mathbf{r} = -4\mathbf{j} + \lambda(2\mathbf{i} + 6\mathbf{j} + \frac{q}{2}\mathbf{k})$ ,  $\mathbf{r} = 4\mathbf{i} + \mu(-6\mathbf{i} - 2\mathbf{j} + \frac{q}{2}\mathbf{k})$ ;

(b)  $\overrightarrow{OU} = -4\mathbf{j} + \frac{1}{2}(2\mathbf{i} + 6\mathbf{j} + \frac{q}{2}\mathbf{k}) = \mathbf{i} - \mathbf{j} + \frac{q}{4}\mathbf{k}$ ;

(c)  $q = 4\sqrt{2}$

# MATHEMATICS (T) (954/2)

## OVERALL PERFORMANCE

The number of candidates for this subject was 3 977. The percentage of candidates who obtained a full pass was 52.45%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	10.28	7.42	6.19	7.67	5.68	8.40	6.81	3.39	3.97	3.75	36.43

## CANDIDATES' RESPONSES

### PAPER 954/2

#### *General comments*

There were instances of candidates still dividing pages into two columns. This caused examiners difficulty in indicating clearly where marks should be awarded, and should be discouraged. Many good and excellent scripts were seen and the standard of presentation was usually good. The paper seemed to give all candidates the opportunity to show what they had learned and understood on a number of questions. Many candidates were able to demonstrate their mathematical ability on this paper. This was a paper which enabled the well prepared candidates to perform well, demonstrating a good understanding of the syllabus content and how to apply the associated skills learned. It was also noticed that some candidates had not done enough preparation and as a result they performed very poorly.

Good candidates were able to plan and gave well-organised answers. They showed all the essential workings. Their handwriting was neat, clear and easily understood. The graph sketched by them was accurate and correctly labelled. They were able to give appropriate descriptions and explanations as requested in the questions. Moderate candidates were able to present their answers well for the questions or the parts they were familiar with. Most of them managed to get the first part or some parts correctly in their workings. They could not answer those questions that require further knowledge and applications of the topics such as Q1, Q2, Q3 and Q7.

Weak candidates with the poor foundations in their basic mathematics concepts, lacked the aptitude and understanding of what were required in the questions. They did not answer according to the requirement of the questions. They could not apply the mathematical concepts to solve the problems. Answers given by them were disorganised and not properly presented. Their presentations reflected their weakness in many aspects, such as not understanding the question, not knowing the concept, not knowing or remembering the formulae, and some even to the extent not able to apply those formulae provided in the question booklet.

#### *Comments on individual questions*

##### **Question 1**

Good candidates were able to eliminate modulus in part (a) correctly. They could factorise and cancel out factor  $-(3 - x)$  correctly. Weak candidates did the wrong elimination of modulus. Some candidates thought that function was always positive when approaching from the positive side. Therefore,  $|3 - x| = 3 - x$  when  $x \rightarrow 3^+$ . Candidates were able to multiply with correct conjugate in part (b). They could recognise  $x^3$  as the highest power of  $x$  from the denominator to carry out the next step of division by  $x^3$ . There were some candidates who could recognise conjugate, only multiplied then conjugated with numerator that led to a different function. For candidates who did not recognise the conjugate, they just straightaway divided

$\sqrt{x^6 + 12x^3} - x^3$  by  $x^3$ . Many candidates did not show  $\frac{12}{x^3} = 0$  as  $x \rightarrow \infty$  when they evaluated the limit, which caused the penalty in this part of the question.

Answers: (a)  $\frac{1}{13}$ ; (b) 6

### Question 2

Most of the good candidates were able to perform product rule and implicit rule correctly. Then, they could obtain the gradient of normal and find equation of normal correctly. A majority of the candidates did not express  $\frac{dy}{dx}$  in the simplest form. Some of the candidates were not able to perform implicit rule correctly

especially for the term  $\sin 2y$ . They differentiated  $\sin 2y$  with missing of 2 or missing of  $\frac{dy}{dx}$ . For the second part of the question, some candidates did not choose the correct value as given in the question.

Answers:  $\frac{dy}{dx} = \frac{-\sin 2y - 2x}{2x \cos 2y + y^2}$ ; Equation of normal:  $y = x + 3$

### Question 3

Good candidates were able to use properties of logarithms correctly and were able to solve the definite integral using integration by parts with correct choice of function. They were also able to use 'hence' to solve  $\int_1^2 \ln x(x+1)^{4x} dx$  using  $\int_1^2 \ln x + 4 \ln(x+1)^x dx$  that led to the exact value of  $2\ln 2 + 6\ln 3 - 2$ .

Candidates without strong concept of logarithms were not able to solve this question. Some candidates were able to show  $\int_1^2 \ln(x+1)^x dx = \frac{3}{2}\ln 3 - \frac{1}{4}$ , but could not evaluate  $\int_1^2 \ln x(x+1)^{4x} dx$  because they

started with the wrong concept of logarithms where  $\int_1^2 \ln x(x+1)^{4x} dx = \int_1^2 4x \ln x(x+1) dx$ . Some candidates did not read the question carefully and gave the final answer as 5.978, which was not accepted. Weak candidates were not able to use the law of logarithm correctly. Hence, they expressed

$\int_1^2 \ln x(x+1)^{4x} dx = \int_1^2 4x \ln x + 4x \ln(x+1)^{4x} dx$ , and end up losing all the marks.

Answer:  $2\ln 2 + 6\ln 3 - 2$

### Question 4

Most of the candidates were able to differentiate  $y = ux$  with respect to  $x$  and then they were able to substitute  $\frac{dy}{dx}$  into the given differential equation and eliminate all  $y$ . Candidates also had no problem to

integrate  $\frac{1}{x}$ . Some careless mistakes occurred for sign error, either left-hand side or right-hand side. For

weak candidates, they were not able to integrate  $\frac{1+u^2}{u}$ . They did not give the final answer in the simplest

equivalent form, for example  $\ln \frac{y^2}{x^2} + \frac{y^2}{2x^2} = -\ln x + \frac{1}{2}$ , where they should simplify the like term. Around 5%

of the candidates still showed the weakness in separating variable and wrote as  $\int \frac{u}{1+u^2} du = -\int \frac{1}{x} dx$ .

Hence, they lose a lot of marks. There were also candidates assumed that  $x \frac{du}{dx} + \frac{u}{1+u^2} = 0$  as the first order linear differential equation. Consequently, they could not solve the ordinary differential equation.

Answer:  $\ln y = \frac{1}{2} - \frac{y^2}{2x^2}$

### Question 5

Many candidates were able to determine the Maclaurin series for  $\cos 2x$  correctly. For those who were able to use the correct trigonometric identities of  $\cos 2x$ , a majority could obtain the answer correctly. There were some careless mistakes done by a few candidates regarding the Maclaurin series. They did not put brackets on  $2x$  while expanding the series, and the candidates stated  $\cos 2x = 1 - \frac{2x^2}{2!} + \frac{2x^4}{4!} - \frac{2x^6}{6!} + \dots$ . For the second part, some candidates did not understand the question with 'hence' and therefore, the candidates used the Maclaurin series of  $\sin x$  to evaluate  $\lim_{x \rightarrow 0} \frac{\sin^2(x) - x^2}{x^4}$ . There were also many candidates who could not remember the trigonometric identities which led to the wrong answer.

Answer:  $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots; -\frac{1}{3}$

### Question 6

Most of the candidates were able to determine  $h$ , the five  $x$ -ordinates and the values of  $y$  correctly, and then substituted into correct trapezium rule to obtain the approximation. Some candidates were not able to find  $h$  correctly, they divided with the number of ordinates instead of  $n - 1$ . Very few candidates used chopping method to find the values of  $y$ . Some candidates forgot to show  $\approx$  at least once. This caused the candidates to be penalised 1 mark for not giving well-presented working. This question was poorly answered by weak candidates because they could not differentiate between ordinate and interval. Hence, they expressed the width of the interval as  $h = \frac{0.8 - (0)}{5} = 0.16$  and lose all the marks for the question.

Answer: 4.839 (3 dp) or  $\approx 4.839$

### Question 7

Some good candidates managed to get full marks for this question. Good candidates were able to determine the correct integrating factor  $e^{\int \frac{x}{4+x^2} dx} = \sqrt{4+x^2}$  and finally could find the equation of the curve as  $y = \frac{x+2}{\sqrt{4+x^2}}$  correctly. Some candidates did not understand the question due to poor concept and tried to

find equation of the curve using  $\frac{dy}{dx} = 0$ . In part (b), candidates who could determine the maximum points

$(2, \sqrt{2})$ , a majority of them determined the nature using the neighbourhood test correctly. Some candidates did not verify the maximum point. They just straight away stated the maximum point after finding  $x$  and  $y$ . For part (c), many candidates could not determine both the asymptotes,  $y = -1$  and  $y = 1$  with complete correct

working using limit  $\lim_{x \rightarrow +\infty} (y) = \lim_{x \rightarrow +\infty} \left( \frac{1 + \frac{2}{x}}{\sqrt{\frac{4}{x^2} + 1}} \right) = 1$  and  $\lim_{x \rightarrow +\infty} (y) = \lim_{x \rightarrow +\infty} \left( \frac{1 + \frac{2}{x}}{\sqrt{\frac{4}{x^2} + 1}} \right) = -1$ , and none of them

expressed in the form of  $y^2$ . Some candidates only stated the equation of asymptotes without any working. A few candidates could get full marks for the graph sketching in part (d) although they could not manage to get full marks for part (c). A majority of candidates were not able to sketch the graph correctly mainly due to only one asymptote found. Hence, the shape of the graph in 3rd quadrant was toward infinity. For some candidates, even though they could sketch the graph correctly, but they were not able to indicate the maximum point and the asymptotes.

Answers: (a)  $y = \frac{x+2}{\sqrt{4+x^2}}$ ; (b)  $(2, \sqrt{2})$ ; (c)  $y = -1$  and  $y = 1$

### Question 8

Many candidates were able to sketch at least one graph correctly. Many candidates could find

$f(x) = 2x + \frac{3}{x-2}$  and then proceeded to use the Newton-Raphson method correctly to obtain full marks.

Quite a number of candidates could find the required area with their limits correctly. Most of the candidates did not notice that the question required the graph in the interval  $[0, \infty)$ . Some candidates ignored the asymptote of the graph  $y = 3\ln(x-2)$  and did not label the graph completely. Many candidates missed out the working in showing how to form the equation  $x^2 + 3\ln(x-2) - 16 = 0$ . A few candidates still used four significant figures in calculating the value of  $x = p$ . Hence, they lose marks as too early approximation. There were candidates still using Newton-Raphson method without showing the working of two iterations. Some candidates wrote "root = 3.778 (4 s.f)", which was not appropriate. Around 25% of the candidates still showed weakness in using Newton-Raphson method correctly. They did not state  $f(x)$  and  $f'(x)$ , did not show the substitution in the first or second iteration, did not state the  $x$ -coordinate instead as root and did not stop at the proper step.

Answers: (b) 3.778, 0.19349

# MATHEMATICS (T) (954/3)

## OVERALL PERFORMANCE

The number of candidates for this subject was 3 944. The percentage of candidates who obtained a full pass was 59.96%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	12.22	9.31	7.20	9.33	7.53	7.30	7.07	5.55	3.35	5.88	25.25

## CANDIDATES' RESPONSES

### PAPER 954/3

#### *General comments*

In general, the performance of candidates was average. They showed a wide range of mathematical abilities and mixed quality of answers were given. Most of candidates were good at answering quantitative questions; Q1(a), Q2(a), Q3(a), Q4(b), Q6, Q8(a) and Q8(b), but were weak in answering conceptual questions; Q1(b), Q2(c), Q4(a) and Q7(b). Most candidates still showed weaknesses in solving problems involving probability such as Q2 and Q7, hypothesis testing in Q5, chi-squared test in Q6 and sampling related question such as Q4. Generally, many poor candidates answered the questions without understanding the concept and the correct way of presentation including proper symbols such as in Q4, Q5 and Q8. Basically, candidates were weak to infer and give comments as seen in Q1(b) and Q8(c). However, Q1(a) and Q6 were well answered and presented. With the usage of scientific calculators, many candidates were not able to perform correct factorisation or using correct formula for solving quadratic equations. High dependency on using calculator to solve questions could be widely seen in candidates solutions. Many candidates also did not show the steps to evaluate the probability of their values in Q4(b)(i), (ii) and 8(b), instead they just wrote the area obtained from the calculator directly.

Good candidates were able to understand the statistic concept well and gave well-organised answers in terms of planning and presentation. They showed all the essential workings accurately and systematically with correct statements as required. Moderate candidates were able to present their answers well for the questions or the parts they were familiar with. Most of them managed to get the first part or some parts correctly in their workings. They showed understanding and competence in answering some or part of the questions. They could not answer those questions that required further knowledge and applications of the topics such as Q1, Q2, Q3, Q7 and Q8. Symbols for  $p$ ,  $p_s$ ,  $\pi$ ,  $\hat{p}$  or  $\sigma$  and  $\hat{\sigma}$  or  $\bar{x}$ ,  $\mu$  and  $\hat{\mu}$  were always non-differentiable by these moderate candidates. Weak candidates with poor foundations in their basic statistical concepts, lacked the aptitude and understanding of what were required in the questions. They did not answer according to the requirement of the questions. They could not apply the mathematical concepts to solve the problems. Answers given were disorganised and not properly presented. Only parts of certain questions were attempted and their answers were either incomplete or incorrect. Their presentations reflected their weaknesses in many aspects, such as understanding the question, not knowing the concept, and not knowing or remembering the formulae.

Candidates also had limited knowledge of the topics with large gaps in understanding. In Section B, about 75% of the candidates chose Q8 instead of Q7. Better performance were seen in candidates that answer Q8, mainly due to candidates who had difficulty when answering questions involving probability.

## Comments on individual questions

### Question 1

All candidates attempted this question. Most of them were able to determine the median and interquartile range. Most candidates could not interpret the data or comment correctly. They just compared the median weight and the interquartile range instead of the weight, hence not able to comment on the weight of sweet potatoes grown using different fertiliser. The majority of the candidates were also not able to give comment on measure of dispersion.

Answers: (a) 102, 24

### Question 2

It was a simple and straight forward question. Most candidates performed well and were able to give correct answers. Some candidates were able to state the probability required directly from the table. Most candidates showed good understanding in using contingency table to evaluate the required probability. Some of the candidates could identify the required conditional probability correctly to the extent that the

good ones were able to use the formula  $P(T_{>40} | T_D) = \frac{n(T_{>40} \cap T_D)}{n(T_D)}$  to state the value directly. Unfortunately,

some students could not apply the correct way to solve it. Some moderate and weak candidates found the probability of intersection of two events in their conditional probability by multiplying the respective event's probability. Majority of the candidates did not understand the requirement of the question. Hence, they could not prove the independency between the two given events. Some candidates could not relate answers obtained in part (a) and part (b) and thus, they were not able to answer this question well. Maybe it was due to the question which did not mention the age group, which caused confusion to the candidates.

Answers: (a)  $\frac{47}{60}$ ; (b)  $\frac{8}{47}$

### Question 3

Only good candidates were able to state that  $F(b) = 1$ . They were able to form equation required and solved accordingly and also stated clearly the value of  $b$  explicitly. Some candidates were able to form correctly the equation as  $2b^2 - 3b + 1 = 1$ , but proceeded writing  $b = \frac{3}{2}$ , without any working. There were a rather

noticeable number of candidates who presented conceptually wrong steps by integrating the function of the given distribution and equated it with 1 to obtain  $b$ . To find the median, candidates should equate the cumulative function to 0.5 and solved the quadratic equation. However, candidates made the same mistake by not showing the correct method to solve the quadratic equation. Most candidates were able to identify

that  $f(x) = \frac{d[F(x)]}{dx}$ , but they were not able to state the limit correctly. Meanwhile, the weak candidates were not able to differentiate the simple quadratic function.

Answers: (a)  $\frac{3}{2}$ ; (b) 1.3090; (c)  $f(x) = \begin{cases} 4x - 3, & 1 < x \leq \frac{3}{2} \\ 0, & \text{otherwise} \end{cases}$

#### Question 4

This part of the question was poorly performed by the candidates. They were not able to apply the conditions for central limit theorem or large sample size and relate it with the normality of a sampling distribution. Many candidates used  $np$  and  $npq$  to obtain the mean and variance, thus resulted the wrong answer. Some candidates were able to obtain standard error correctly but not able to use correct symbol to represent the distribution as  $\hat{p} \sim N(0.317, 0.0014434)$ , instead they used  $p$ ,  $x$  or  $\mu$ . Many candidates were able to perform standardisation correctly to find the  $Z$  value. Some candidates gave the correct probability without stating the correct method to find the probability ( $1 - P(Z < 2.185)$ ). Some candidates used the wrong symbol for the sample proportion, such as  $P(p > 0.4)$ ,  $P(\bar{X} > 0.4)$  or  $P(X > 0.4)$ . Most candidates were not able to state  $P(|\hat{p} - p| \leq 0.10)$  or  $P(-0.10 \leq \hat{p} - p \leq 0.10)$  for the given probability. Hence, the performance of candidates for this portion of the question was very poor.

Answer. (a)  $\hat{p} \sim N(0.317, 0.0014434)$ ; (b) 0.0144; (c) 0.9914

#### Question 5

Most candidates were not able to answer this question correctly. They did not carry out the test for a small sample using a binomial distribution, but they assumed sample proportion followed a normal distribution. Many candidates did not give correct null and alternative hypotheses, such as  $H_0: \hat{p} = 0.8$ ,  $H_1: p > 0.8$ . A few candidates still gave hypotheses in sentences, which were not accepted. Some candidates even compared the critical value obtained from the table with the value of probability found. This shows that candidates did not master this chapter well.

Answer: -

#### Question 6

Candidates answered this question well. They ought to use the Poisson distribution formula to find the probability and the expected values. Majority of the candidates were able to calculate the chi-squared value and were able to compare and made correct conclusion. However, there were a handful of candidates that worked on two significant figures for the values of probability obtained. There were some candidates who did not consistently work their answers at three or more significant figures. Some candidates did not combine the last two adjacent classes where the expected frequency was less than 5 to obtain new  $E_i$  which led to the wrong degree of freedom and the critical value. Only the weak candidates were not able to solve this question.

Answer: -

#### Question 7

This question were quite direct and easy to answer. Since probability question was not that popular, majority of the candidates did not choose this question. However, those who attempted this question could answer part (a) correctly. For part (b), they were not able to state the distribution correctly. Many candidates gave the wrong answer such as a normal distribution or a binomial distribution with  $n = 7$  and  $p = \frac{3}{7}$ . Most of the candidates did not realise that they should use the binomial distribution to solve. Even though some candidates could identify the distribution as binomial, but they assumed  $n = 7$  (total number of balls), which led to the wrong answer for  $E(X)$  and  $\text{Var}(X)$ . Good candidates who were able to identify the distribution as binomial distribution, could find the mean and variance easily by using the formula  $E(Y) = np$  and  $\text{Var}(Y) = npq$ .

Answer: (a)(ii)  $\frac{9}{7}, \frac{24}{49}$ ; (b) (i)  $Y \sim B_{\frac{3}{7}, \frac{3}{7}}$ , (ii)  $\frac{9}{7}, \frac{36}{49}$

### Question 8

Most candidates could apply the correct formula in part (a) for the calculation to obtain the value correctly. The weak candidates used the wrong symbol. In part (b), candidates did not use the correct sign for the sample mean and the correct inequality. Most of them could carry out standardisation correctly but around 20% of the candidates did not use the correct method for evaluating the probability. They were not able to show clearly the use of lower tail probability to obtain  $P(Z \geq -1.8590)$  but instead calculator was used to give the answer directly. A small number of candidates still applied continuity correction before standardisation was carried out, which was not accepted. Candidates used the correct  $Z$  value in the correct formula and obtained the correct confidence interval in part (c). Almost all the candidates were not able to interpret correctly the answer obtained from the confidence interval. For part (d), many candidates were able to show the working to find the width of confidence interval from the information given and formed an equation to solve for  $n$ . The candidates need to state "the minimum sample size equals 35" but some only state  $n = 35$  which could not be accepted.

Answer: (a)  $\frac{572}{5}$ ; (b) 0.9684; (c) (112.28, 116.52); (d) Minimum sample size = 35