MAJLIS PEPERIKSAAN MALAYSIA
Malaysian Examinations Council

PELANGI

Laporan Peperiksaan

STPM 2023

Matematics (T) (954)

Matematics (T) 954/1

OVERALL PERFORMANCE

The number of candidates for this subject was 3 047. The percentage of candidates who obtained a full pass was 42.30%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	6.56	3.25	2.46	4.76	7.52	6.99	10.76	5.54	8.77	4.99	38.40

RESPONSES OF CANDIDATES

PAPER 954/1

General comments

All questions were answered by nearly all the candidates in English. Generally, most candidates made an attempt at all of the questions which suggests that the questions were "do able". Most of them answered sequentially the questions through the paper. Again, illustrating no difficult questions that made them to skip any question. Quite a significant number of candidates arranged the answers in proper order and managed to write well-presented and neat answers but some candidates still used two columns in presenting their working, which make it hard for the examiners to write marks at the appropriate place. Quite a number of the candidates produced incoherent and disorganised answers, which showed unplanned working and these illustrates lack of practice.

In terms of the standard of solutions/answer given, the candidates presented mixed quality of answers with a wide range of mathematical abilities. Majority of the candidates answered well for questions which uses direct application of mathematical methods with elementary skills. For candidates who depend too much on the usage of calculators, the answers given were not accepted. Candidates just stated the answers without showing essential steps to convince as to how the answers came about.

In Section B, most candidates answered only one question as instructed. Question 7 was the more favourable choice compared to question 8. Better performances were seen in candidates that answer question 7 than in question 8, mainly due to candidates do not have the acquired skills and the depthness of understanding to enable them to think critically using proper reasoning and justifications in such questions relating to functions of question 8.

Good candidates were able to plan, produced well-organised answers with neat and clear handwritings. Essential workings were nicely provided. The graphs were accurately sketched and correctly labelled. The moderate ones were able to present their answers well for the questions or the parts they were familiar with. Most of them managed to get the first part or some parts correct in their workings. They could not answer those questions that require further knowledge and applications of the topics such as questions 1, 3, 4 and 8.

Weak candidates had poor foundations in their basic mathematics concepts, lacked the aptitude and understanding of what were required in the questions. They did not answer according to the requirement of the questions. They could not apply the mathematical concepts to solve the problems. Answers given were disorganised and not properly presented. Mostly, they failed to solve the questions involving the instruction of "hence". This can be seen in question 3, 4, 7, and 8. Their presentations reflected their weakness in many aspects, such as not understanding the question, not knowing the concept, unable to apply the provided information given in the question booklet.

Comments on individual questions

Question 1

This question required the candidates to solve the simultaneous equation provided. Knowledge of exponent and logarithm properties are essential to solve this question. This question is not that difficult for candidates with basic knowledge in logarithm. Nevertheless this question is considered as poorly answered. Candidates are unable to obtain the correct answer even though they managed to get the correct quadratic equation due to incorrect way of solving. Very few candidates obtained full marks. Most of the candidates were able to use the properties of exponents to obtain the correct relation between x and y. Few of them were able to apply the properties of logarithms, $\log_x 4x = \log_x 4 + \log_x x$ correctly. Most candidates who substituted the relation of xy and were able to apply the properties of log correctly; successfully solved for x value and attained the value of y. Majority of the candidates were able to substitute $xy = 4$ into the second equation but failed to obtain values of x and y because of the incorrect law of logarithms used. Quite a number of candidates managed to get the correct quadratic equation but unfortunately solve the quadratic equation wrongly while some did not justify the values of x obtained. Candidates' lack of knowledge on the properties of logarithms and writing all sort of relation led them to wrong answers. One weakness in applying the properties of logarithms when changing the base of logarithm, i.e $\log_x 4 = \dfrac{\log_4 4x}{\log_4 x} \Rightarrow \log_4 (4x - x) \Rightarrow \log_4 3x$. Another familiar mistake exhibited in the workings i.e $\log_x 4x = 2 \Rightarrow x^2 = 4x \Rightarrow x = 4$; using cancellation of x which ended up losing of marks. There were also a handful of candidates that did not obtain full marks due to not realising that $x = 0$ need to be rejected since $x > 0$.

Answers: $x = 4$; $y = 1$

Question 2

A good number of candidates could answer the first part superbly. Since this question is a familiar one, candidates took the chances to score marks as much as possible. On the other hand, the second part was not contributing marks to the average and weak candidates since only few candidates successfully stated the valid range of x. Most candidates excellently completed this task by giving the correct expansion using the binomial theorem in expanding $(1 + 2x)^3$ and $(1 - x)^{\frac{1}{2}}$ separately. Candidates multiplied both series appropriately and successfully continue to find the range of x, knowing that the expansion of $\dfrac{(1 + 2x)^3}{\sqrt{1 - x}}$ was valid when $|-x| < 1$ and concluded that $-1 < x < 1$. Quite a number of the candidates were able to expand $(1 + 2x)^3$ correctly but presented the expansion inappropriately by writing '+ ...' at the end of the expansion whilst some did not write '+ ....' at all for the expansion of infinite series $(1-x)^{\frac{1}{2}}$ or in evaluating $\dfrac{(1+2x)^3}{\sqrt{1-x}}$ hence their marks were penalised. Only few candidates were able to state the range of values of x correctly. Candidates did not realise that the expansion of $(1+2x)^3$ is valid for all real values of x. Consequently, most candidates wrote that the range of validity was $-\frac{1}{2} < x < \frac{1}{2}$ and $-1 < x < 1$; hence obtaining $-\frac{1}{2} < x < \frac{1}{2}$ as the range of values of x. Quite a number of candidates attempted the second part and managed to state the valid range of values correctly; $-1 < x < 1$ however, candidates did not mention that $|-x| < 1$. A '*no essential working*' mark was deducted. Few candidates tried to expand $(1-x)^{\frac{1}{2}}$ instead of $(1-x)^{-\frac{1}{2}}$ and left the answer as $\dfrac{(1+2x)^3}{(1-x)^{\frac{1}{2}}} = \dfrac{1+6x+12x^2+8x^3}{1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^2+\ldots}$. No long division seen being performed in the attempt to get the expansion requested.

Answers: $1+\frac{13}{2}x+\frac{123}{8}x^2+\frac{265}{16}x^3+\ldots$; $-1 < x < 1$ or $(-1, 1)$

Question 3

Matrix question is a favourite topic almost every year. The question consisted of two unknowns as entries in the matrix resulting in low performance in this question. Candidates were unable to deal with unknowns while performing the elementary row operation (ERO). Almost all candidates were able to write the augmented matrix correctly and use the ERO to obtain 0's at entries (2, 1), (3, 1) and (3, 2) based on their operations. The first three marks were easily achieved. Even though candidates obtained the wrong f(k), their approach to solve the question by equating their f(k) = 0 and entry (3, 4): $m + km - 7k + 7 = 0$ indicating that they understood the requirement for 'infinitely many solutions of a system of equations'. With their values of k and m, they were able to use correct approach to obtain the general solution by equating either the variable of y or z with a parameter and proceeded with finding the other two variables even though their answers were wrong. Candidates were able to write the augmented matrix correctly and utilised correct ERO to obtain 0's at (2, 1), (3, 1) and (3, 2), but unsuccessful in producing correct entries at other position, and hence, the f(k) obtained was incorrect. Some candidates still showed weaknesses in writing proper ERO instructions and did mistakes during the algebraic operation on the entries.

Answers: $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & k-1 & -1 & m \\ 0 & 0 & k^2-k-2 & m+mk-7k+7 \end{array}\right)$; $f(k) = k^2 - k - 2$

$k = -1$(rejected); $k = 2$; $m = \frac{7}{3}$

$x = \frac{7}{3}$; $y = \frac{7}{3} + t$; $z = t$

Question 4

The question was a 'hence' question. The answer in the second part relied on the verification of the rootspecified in the first part. Quite a number of the candidates skipped the first part and proceeded with the second one. Even though candidates successfully attained the correct answer in the second part, the last mark could not be granted. Candidates were capable of substituting $z = 3\text{i}$ into the equation and clearly showed the working to attain 0, utilising $\text{i}^2 = -1$; $\text{i}^3 = -\text{i}$; $\text{i}^4 = 1$. Most candidates knew that the conjugate of 3i i.e −3i was a root too. With this information, candidates excellently found the second quadratic factor using long division method or comparing of coefficients. Using the formula $z = \frac{-\text{b}\pm\sqrt{b^2 - 4ac}}{2a}$, candidates succeeded in getting the other two roots. Few candidates straight away used the calculator to find roots and stated all the roots of the equation without any working shown. A handful of candidates attempted this question without verifying that 3i was a root of the equation. Some candidates substituted $z = 3\text{i}$ in the equation; $4\text{z}^4 - 12\text{z}^3 + 49\text{z}^2 - 108\text{z} + 117 = 4(3\text{i})^4 - 12(3\text{i})^3 + 49(3\text{i})^2 - 108(3\text{i})^2 + 117 = 0$, without showing the steps that led to zero. Hence, 'no essential working' mark was deducted. Some candidates used polar form to solve the problem by finding the argument and magnitude of the first root. A small number of candidates were incapable of deducing that $z = -3\text{i}$ was another root of the equation. Due to this, candidates were unable to obtain the two quadratic factors which led to the final answers.

Answers: $z = 3\text{i}; -3\text{i}; \frac{3}{2} + \text{i}; \frac{3}{2} - \text{i}$

Question 5

This question was remarkably done by majority of the candidates. Almost all candidates attempted this task and attained at least the first three marks. Most candidates were able to show that the parametric equation, $x = 2 + 2\cos\theta$ and $y = -1 + 3\sin\theta$ could be represented in the Cartesian form as $\frac{(x-2)^2}{2^2} + \frac{(y+1)^2}{3^2} = 1$. Most candidates easily attained the Cartesian form by eliminating θ using trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ to form the equation $\frac{(x-2)^2}{2^2} + \frac{(y+1)^2}{3^2} = 1$. Candidates brilliantly concluded that the conic was an ellipse. The standard form of ellipse was written as $\left(\frac{x-2}{2}\right)^2 + \left(\frac{y+1}{3}\right)^2 = 1$ by a handful of candidates. However, these candidates generally were able to state the conic as an ellipse. Few candidates were unable to recognise the conic as an ellipse even though the standard equation was given in the question booklet. A handful of the candidates were able to find the coordinates of the centre and vertices correctly but failed to sketch a vertical ellipse. They instead sketched ellipses with shapes that looked like circle or diamond even though they were able to indicate the centre and vertices correctly. Quite a number of them were penalised for the poor graphs sketched. Few candidates were unsuccessful in obtaining full marks due to the ellipse sketched did not touch the y-axis at $(0, -1)$.

Answers: (*a*) $\frac{(x-2)^2}{2^2} + \frac{(y+1)^2}{3^2} = 1$, Ellipse

Question 6

Most of the candidates used the substitution correctly. However, quite a number of candidates did not conclude, which caused losing a mark. Many of the candidates were also confused on finding the angle between the line and plane. Quite a number instead found the angle between the line and the normal of the plane. Most candidates were able to show that the point Q lies on the plane μ. Mostly substituted (−1, 1, 1) in plane equation, $3x - 2y + z = 3(-1) - 2(1) + (1) = -4$. Some candidates used the dot product: $\overrightarrow{OQ}\cdot\mathbf{n} = \begin{pmatrix}-1\\1\\1\end{pmatrix}\begin{pmatrix}3\\-2\\1\end{pmatrix} = -3 - 2 + 1 = -4$. Candidates were able to find direction $\overrightarrow{PR}$ or $\overrightarrow{RP}$ correctly and subsequently attained the vector equation of the line, l. Quite a number of candidates successfully determined the angle between the line and plane using either the sine or cosine formulae. For the last part of the question, candidates who obtained the correct equation by substituting line equation into plane y = 0, successfully used the correct value of the parameter achieved and managed to find the point of intersection. A number of candidates directly wrote without showing the operations of the dot product i.e $= \begin{pmatrix}-1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}3\\-2\\1\end{pmatrix} = -4$. Hence, the mark was penalized with a '*No essential working*' remark. A small number of candidates used cross product or dot product to find $\overrightarrow{PR}$ or $\overrightarrow{RP}$. A handful of candidates used the scalar notation for vector equation whereby they wrote l instead of $\bar{r}$. For candidates who used the sine formula, $\sin\theta = \frac{\underline{n}\cdot\underline{b}}{|\underline{n}|\ |\underline{b}|}$, the answer required them to define θ as the angle between the line and the plane. Quite a number of candidates failed to do so and end up losing the two marks. While for some of candidates who utilised the cosine formula; $\cos\alpha = \frac{\underline{n}\cdot\underline{b}}{|\underline{n}|\ |\underline{b}|}$, they missed out the answer due to not substracting the attained angle with 90°. Candidates misunderstood and assumed that α was the angle between the line and the plane. Majority of the candidates failed to identify that the plane $y = 0$ was $0x + y + 0z = 0$. Most of them substituted the x, y and z values into the given plane equation $3x - 2y + z = -4$, thus they lost all the marks. Many candidates applied correct concept but used wrong plane to find the point of intersection between l and the plane $y = 0$. Quite a handful of candidates wrote the point of intersection in vector form to represent the point required by the question. Most candidates only managed to score four marks because mostly they failed to answer part b(ii) and b(iii).

Answers: (b) l: $\mathbf{r} = \begin{pmatrix}3\\2\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}4\\1\\5\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$; ($c$) $\theta = 40.14°$; (d) (5, 0, 8)

Question 7

Most of the candidates who chose this question successfully attained the first three marks. Parts (*a*) and (*b*) were straight forward and majority of those who attempted this question did not do part (*c*) and (*d*) well. Almost all candidates who attempted this question were capable of expressing $2 \sin \theta + 2\sqrt{3} \cos \theta$ in the form $R \sin(\theta + \alpha) = 4 \sin \left(\theta + \frac{\pi}{3}\right)$. Candidates were able to state that $-1 < \sin \left(\theta + \frac{\pi}{3}\right) \leqslant 1$ and successfully form $-4 < 4 \sin \left(\theta + \frac{\pi}{3}\right) \leqslant 4$. Hence, they could conclude the maximum and minimum values for $2 \sin \theta + 2\sqrt{3} \cos \theta$. Quite a number of candidates gave good sketching with correct shape (with one maximum point, one minimum point dan two x-intercepts) between $-\pi < \theta \leqslant \pi$. The indication of those things were clearly shown. Almost all candidates who attempted this question managed to achieve $\sin \left(\theta + \frac{\pi}{3}\right) = 0$ and pursued to get the correct values for θ. Quite a few candidates stated the maximum and minimum values correctly without any justification, $(-1 \sin \left(\theta + \frac{\pi}{3}\right) \leqslant 1)$, therefore, the mark was deducted. Candidates also stated correctly that $\theta + \frac{\pi}{3} = 0, \pi, 2\pi, -\pi$ without realising that $-\pi < \theta \leqslant \pi$. They left the answer as $\theta = -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, -\frac{4\pi}{3}$, which was incorrect. Few candidates who showed good sketching of the graph lost the last sketching mark due to not indicating the end point at $x = -\pi$ with an open circle. Only few candidates managed to solve the inequality of the expression given to find the range of values.

Answers: $4 \sin \left(\theta + \frac{\pi}{3}\right)$;

(*a*) Min. value = −4, Max. value = 4;

(*b*) $\theta = -\frac{\pi}{3}, \frac{2\pi}{3}$;

(*c*) $\left(-\frac{\pi}{2}, -\frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$

Question 8

The question was considered as above average in terms of difficulty judging by the performance and popularity of this question. Part (*a*) was a straight forward but for part (*b*) only a handful of candidates were able to see the relation between the results obtained in (*a*) and how to utilise the result to answer part (*b*). Almost all the candidates who attempted this question were able to express $4r^2 - 1$ as two linear factors and attained $\frac{1}{4r^2 - 1} \equiv \frac{A}{2r - 1} + \frac{B}{2r - 1}$. These candidates proceeded to find A and B and ended up getting the partial fractions correctly. Candidates managed to find the sum of the first n terms using summation sign with partial fraction and proceeded with method of differences. The expansion for at least the first three terms and last two terms were clearly shown. Few candidates used the formula and managed to get the exact answer. Most candidates who successfully attained the sum of the first n terms of the series, accurately exhibited the sum to infinity of the series. Candidates who used calculator to find the prime factor of the denominator, ended up expressing $\frac{1}{4r^2 - 1} \equiv \frac{A}{r - \frac{1}{2}} + \frac{B}{r - \frac{1}{2}}$ or $\frac{1}{4r^2 - 1} \equiv \frac{A}{r - 1} + \frac{B}{r + 1}$. Performance of the candidates for this part of the question was moderate as some could not even write the sum of first n term of the series using summation sign with the partial fraction they obtained in the previous section. Quite a number of candidates who got the correct partial fractions and used the method of differences to find the sum of first n terms stated only the first two pair of terms and the last one pair. Therefore, marks deducted for incomplete answer. Few candidates tried to use arithmetic progression in the attempt to get the sum of first n terms of the given series. Most candidates who attempted this question did not try to solve part (b) of the question except for few candidates who were able to at least express the given infinite series using correct sigma notation and general term as $\sum_{r=1}^{\infty} \frac{1}{r^2}$. Candidates were not able to solve this question because of the failure to get the general term of the given series and related it with $\frac{1}{4r^2 - 1}$.

Answers: (a) (i) $\frac{1}{4r^2 - 1} = \frac{1}{2(2r - 1)} - \frac{1}{2(2r + 1)}$; (ii) $S_n = \frac{n}{(2n + 1)}$; $S_\infty = \frac{1}{2}$

(b) $1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots = \sum_{r=1}^{\infty} \frac{1}{r^2}$

$= \sum_{r=1}^{\infty} \frac{1}{r^2} < 2$

Matematics (T) 954/2

OVERALL PERFORMANCE

The number of candidates for this subject was 2 877. The percentage of candidates who obtained a full pass was 40.29%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	2.57	2.22	3.34	4.87	6.81	10.60	9.87	5.10	3.39	5.10	46.12

RESPONSES OF CANDIDATES

PAPER 954/2

General comments

Generally, this paper seemed to give all candidates the opportunity to show what they had learned and understood on a number of questions. Many candidates were able to demonstrate their mathematical ability in this paper. The paper enabled the well prepared candidates to perform excellently, demonstrating a good understanding of the syllabus content and apply the associated skills learned. It was also noticed that some candidates had not done enough preparation and as a result, they performed very poorly. This was obviously seen in Q4 about trigonometry integration and Q3 which involved two times by part integration.

Most answers to Q1, Q2 and Q5 were done well. Unfortunately, Q3 and Q4 were performed poorly involving part by part integrations in Q3, and Q4 that involved trigonometry integration. Candidates showed weaknesses in choosing correct function to facilitate the integration. There were still quite a number of candidates unable to use integrating factor to solve the differential equation in Q4. Meanwhile, performance of candidates for obtaining Maclaurin series in Q5 was very good compared to previous years. For Q6, Q7 and Q8, candidates performed moderately.

For Section B, overwhelm majority of candidates preferred to answer Q7 which involved application of differentiation including finding extremum and inflexions points, whereas Q8 involved the application of integration to find area and volume. Only a very small amount of candidates chose to attempt Q8. Most of candidates failed to sketch the required graph.

Comments on individual questions

Question 1

Most of the candidates were able to identify correct function. Many candidates used the correct concept of limit by using correct function to simply, evaluated the limit from right and then equalised it to get the value of α. Some moderate and most of the weak candidates did not know how to simplify limit from right by using conjugate or factorisation. There were candidates which still missed to write the limit notation.

Answers: $-6 - \sqrt{3}$

Question 2

Majority a number of candidates were able to use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ and to find the equation of the tangent that was parallel to y-axis. However, some candidates still did not know how to differentiate $\frac{1}{t}$. They were weak in understanding the meaning of the equation of "the tangent that is parallel to y-axis" which mean denominator of $\frac{dy}{dx} = 0$ and not the equation of $\frac{dy}{dx} = 0$. Can get two values of t (do not eliminate 1 value) but candidates find values of y and use equation of straight line to get the equation of the tangent that is parallel to y-axis.

Answers: $\frac{dy}{dx} = \frac{4t^2 + 2}{t^2 - 4}$; ($b$)(i) $x = 2$

Question 3

The good candidates knew how to use integration by parts by repeating the methods for three times. They know correctly about suitable trigonometry identity of this questions. Examples: $1 + \cos^2 3x = \frac{1}{2}(3 + \cos 6x)$ $\sin 6x = 2 \sin 3x \cos 3x$. Even though the majority of weak candidates knew how to use the method of integration by parts but they did not know the correct trigonometry identities to be used and also how to solve with Integration. They were not sure of choosing the values of u and v' while solving the integration by parts. Some candidates wrote wrongly $1 + \cos^2 3x = \sin^2 3x$.

Answers: $\frac{8}{15}e^{\pi} - \frac{8}{15}$

Question 4

Majority of the candidates were able to find the equation $y = f(x)$ with integrating factor of $e^{\sin x}$. They knew how to do the integration by parts and substituted the values of x and y given. For weak candidates, they got the wrong integrating factor ($e^{-\sin x}$) and wrong choice of u and v'.

Answers: $y = \sin x - 1 + 2e^{-\sin x}$

Question 5

Most of candidates were able to apply chain rule and product rule correctly and differentiated the given function implicitly to get and show the answer. Candidates' knowledges of Maclaurin series quite good. There were quite number of candidates differentiated the function $\sqrt{4 + \sin 2x}$ improperly. Some candidates did not know how to find $\frac{d^2y}{dx^2}$ after successfully differentiated the first derivatives. There were also candidates made mistakes of coefficients sign (positive or negative), and hence the Maclaurin series produced a wrong answer.

Answers: (*a*) $2 + \frac{1}{2}x - \frac{1}{16}x^2 - \frac{61}{192}x^3 + \ldots$; (*b*) $\lambda\ -\frac{1}{6}$

Question 6

Most of the candidates were able to find the root by using Newton-Raphson method. They were able to get first two marks and lost the next marks because failed to differentiate $(1 + 3^{-x})$ correctly. Some candidates wrongly multiplied $(1 + 3^{-x})(x^2 + 4) = x^2 + 4 + 3^{-x}\ x^2 + 12^{-x}$, where the term 12^{-x} was obviously wrong. Another weakness which was done by candidates while finding the 'stopping criteria' in the approximation of root to the equation.

Answers: Root = 3.411 (4 sf)

Question 7

The good candidates were well-versed about the concept of the function increases and decreases, extremum point, concave upward and concave downward, and the coordinates of the point of inflexions. They could sketch the graph very well. A few candidates forgot to write x-intercept and y-intercept in their answer. Some candidates could get extremum or minimum point but they failed to get the correct intervals of the function increases and decreases, concave upward and concave downward, and the coordinates of the point of inflexions.

Answers: (*a*) x-intercept = 2.4113 and 3.5887; y-intercept = 0.5000;

(*b*) Increase = $(3, \infty)$, Decrease = $(-\infty, 3)$, Extremum = $\left(3, -\frac{1}{2}\right)$;

(*c*) Concave upward = $\left(\frac{5}{2}, \frac{7}{2}\right)$, Concave downward = $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$

Inflexions = $\left(\frac{5}{2}, -0.10653\right)$ and $\left(\frac{7}{2}, -0.10653\right)$

Question 8

The good candidates were able to sketch the graph of two functions given, and hence, they could identify the area and also the volume by using correct limits and correct formulae of integration. They had no problem while integrating the improper fraction. Some candidates were not able to find the points of intersection correctly. Therefore the limits of finding area and volume were also wrong. A few candidates wrote the wrong formulae of area and volume, where they forgot to square the function while finding the volume, which resulted the wrong answer. There were also candidates ignored π for the formula of volume.

Answers: (*b*) $4 - 3\ln 3$; (c) $\left(16 + 12\ln\frac{1}{3}\right)\pi$

Matematics (T) 954/3

OVERALL PERFORMANCE

The number of candidates for this subject was 2 852. The percentage of candidates who obtained a full pass was 69.28%.

The achievement of candidates according to grades is as follows:

Grade	A	A–	B+	B	B–	C+	C	C–	D+	D	F
Percentage	12.69	10.59	8.27	11.19	8.98	10.06	7.50	6.63	3.58	4.98	15.53

RESPONSES OF CANDIDATES

PAPER 954/3

General comments

Generally, candidates were good at answering quantitative questions (Q1(*a*), Q1(*b*), Q2, Q3, Q4(*a*), Q4(*b*) and Q5) but weak in answering interpretation questions (Q1(*c*), Q8(*c*) and Q8(*e*)). As expected, good candidates performed well and gave well-organised answers in terms of planning and presentation since they were able to understand the statistics concept clearly. They showed all the essential workings accurately and systematically with correct statements as required. Moderate candidates managed to get the first part or some parts of the working correctly. They were able to present their answers well for the questions or the parts they were familiar with. As usual, weak candidates with poor foundations in their basic statistical concepts lacked of the understanding of what were required in the questions. Answers given were disorganised and not properly presented and their answers were either incomplete or incorrect.

Overall, most of the candidates performed well in Section A and not doing well in Section B. For Section A, good and moderate candidates could answer well for most of the questions except Q1(*c*) and Q4(*c*). For Q1(*c*), almost all candidates could not interpret how a wrongly recorded value affected the data. Somehow, the candidates found difficulties in making justification on the data when error occured. For Q4(*c*), majority lost marks when they used confidence interval instead of the width to form the equation to proceed in finding α.

Most candidates chose to answer Q8 instead of Q7 in Section B, even though both questions show similar level of difficulties. Like previous years, there were occasions where candidates used incorrect symbols in their workout solutions which led to loss of some marks. This indicated that they did not understand the meaning and usage of statistical symbols in their workout solutions. With the usage of scientific calculator, several candidates skipped the steps in evaluating probability using the lower-tailed based on normal distribution table attached in the examination questions paper, and they just wrote the solutions obtained from calculator.

Comments on individual questions

Question 1

Almost all candidates attempted and were able to find Q_1, Q_2 and Q_3 since this is an easy and straight forward question. However, a few still showed weaknesses in obtaining the quartiles by assuming $Q_1 = \frac{x_7 + x_8}{2}$ and $Q_1 = \frac{x_{19} + x_{20}}{2}$. In most cases, candidates could use the correct formula to find the fences and also to determine the outlier correctly. Most of them could draw the box-and-whisker plot on graph paper by indicating all the values and the outlier. Some candidates were not able to detect an outlier using he correct formula, hence they were not able to indicate it on the plot. Almost all the candidates did not answer part (*c*) correctly. They knew that value 118 will affect Q_3 but they failed to give a reason why it was affected. This showed that most of candidates were very weak and not able to answer questions concerning explanation, or comments based on data or results obtained.

Answers: (*a*) $Q_1 = 104$, $Q_2 = 115$, $Q_3 = 126$

Question 2

For part (*a*), most candidates could find the probability correctly. They showed good understanding in using formula to evaluate the required probability. They used either the relation $P(A \cap B) = P(A) - P(A \cap B')$ to find $P(A \cap B)$, or they attempted to find $P(B)$ first then used $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to evaluate $P(A \cap B)$ easily. For second part of part (*a*), candidates used $P(B) = P(A \cup B) - P(A \cap B')$ to find $P(B)$. Then use the relation $P(A|B) = \frac{P(A \cap B)}{P(B)}$ to find $P(A|B)$. For part (*b*), some candidates did not show their working clearly or just provided their answer by memorising the general formulae of the mutually exclusive and independent concepts. To show the events were independent, some candidates only stated $P(A) \times P(B) = P(A \cap B)$ without showing the appropriate calculation. Many candidates were able to state A and B were not mutually exclusive by giving reasoning that $P(A \cap B) \neq 0$. However, there were candidates who gave wrong reason such as $P(A \cup B) \neq 0$. Some candidates were able to state A and B were not independent by giving reasoning that $P(A|B) \neq P(A)$ or $P(A|B) \neq P(A)$ or $P(A) \times P(B) = \frac{1}{2} \times \frac{9}{14} \neq P(A \cap B)$. Mostly used the latter option. However, there were candidates who made mistake by wrongly stated that $P(A \cup B) \neq P(A) \times P(B)$ or $P(A|B) \neq P(A) \times P(B)$. The set operation could also be represented using Venn diagram to solve this question. Some candidates used the concepts of conditional probability correctly with the help of Venn diagram.

Answers: (*a*) (i) $\frac{2}{7}$; (ii) $\frac{4}{9}$

Question 3

Only good ones could easily score full marks for this question. Most candidates could only answer part (*a*) correctly. Some of the candidates failed to differentiate between discrete random variable and continuous random variable. They used integration instead of summation to find the value of k. Many candidates could not eliminate the modulus sign with correct inequality and some candidates failed to solve part (*b*) due to not able to eliminate the modulus into correct form, $|x| < a \Leftrightarrow -a < x < a$.

Hence, they were not able to solve $P(|X-2| \leqslant 1)$. A few candidates were able to eliminate the modulus sign, but they applied the incorrect inequality sign which made them ended up with wrong answer. However, those candidates who were able to obtain the probability $P(1 \leqslant X \leqslant 3)$ made mistakes by assuming $P(1 \leqslant X \leqslant 3) = P(X \leqslant 3) - P(X \leqslant 1)$ or $P(1 \leqslant X \leqslant 3) = P(X \leqslant 3) - P(X \geqslant 1)$ or $P(1 \leqslant X \leqslant 3) = P(X = 3) - P(X = 1)$, which supposed to be $P(1 \leqslant X \leqslant 3) = P(X \leqslant 3) - P(X < 1)$.

Answers: (*a*) $k = \frac{1}{12}$; (*b*) $\frac{1}{3}$

Question 4

Candidates have no problems answering part (*a*) and part (*b*). Most candidates who attempted this question used the correct symbol and formula for the unbiased estimator. However, as expected, poor candidates used wrong formula and/or symbols. Those who attempted this question were able to find the unbiased estimate for the mean, the value of $z_{\frac{a}{2}}$ and the standard error correctly, and plug in these values in correct formula to obtain the required confidence interval. Majority gave the answers correct to three decimal places as required by the questions in part (*a*) and part (*b*). Some candidates did not understand the requirement of the question; they only found the sample variance using $s^2 = \frac{6320}{60} - \left(\frac{612}{60}\right)^2 = \frac{97}{75}$ and they failed to find unbiased estimates for variance which supposed to be $\hat{\sigma}^2 = \frac{n}{n-1}(s^2)$. Many candidates wrongly interpreted the requirement of the question and got confused with the meaning of interval width. They used the confidence interval instead to answer this part, i.e. they compared the lower and upper intervals to solve for the confidence level and ended up using the limits wrongly as $10.2 - z_{\frac{a}{2}}\sqrt{\frac{1.315}{40}} = 9.956$ or $10.2 + z_{\frac{a}{2}}\sqrt{\frac{1.315}{40}} = 10.444$ to determine the value of $z_{\frac{a}{2}}$.

Answers: (*a*) 1.315; (*b*) (9.956, 10.444); (*c*) 82.08%

Question 5

Most candidates performed well for this question, even obtained the full marks. They could write correct hypothesis and determined the values of z correctly. Only the weak candidates were not able to answer this question. However, there were some candidates who stated wrong alternative hypothesis, $H_1 : \mu < 15.2$. Hence, they gave the wrong conclusion; even though they obtained the correct value of test statistic . A certain number of candidates wrote the conclusion not in proper way, they missed out the word "sufficient/insufficient" or "at 5 % significance level".

Question 6

Since this was a straightforward and popular question, even the moderate candidates could solve this question well. Most of the candidates were able to obtain full marks. They stated the hypotheses correctly and were able to find the expected frequencies correctly. Hence, they were able to do the test systematically with the correct approach. However, weak candidates still make some common mistakes such as; stated the hypotheses inappropriately, calculated wrongly the degree of freedom, failed to make a decision on their finding, even though they correctly compared their test statistic and critical value, and they could not give precise conclusion with the term '5% significance level' or 'insufficient evidence'.

Question 7

Not many candidates attempted this question. Those who did, mostly could not perform well. Majority who tried this question were able to convert the given mathematical statement to $P(X < 150) = \frac{30}{500}$ and standardised accordingly, and were able to obtain the value of μ correctly. However, even though the weak candidates could reach until $P\left(Z < \frac{150 - \mu}{4.5}\right) = 0.06$, they could not find μ because of their weakness in finding z-score correctly. There were a substantial number of weak candidates that worked meaninglessly and simply gave a value for μ. Majority of candidates did not understand what is meant by 'falls within one standard deviation of the mean' as $P(157.0 - 4.5 \leqslant X \leqslant 157.0 + 4.5)$. Some candidates who tried part (*c*)(i) of the question were able to show the random variable as binomial distribution and used np to find the expected value. Many candidates failed to relate the probability they obtained in part (*b*) with part (*c*)(i), hence failed to answer this part. Even though candidates knew that they must use normal approximation in part (*c*)(ii) but they failed to obtain mean and variance correctly because of mistakes they made in parts (*b*) and (*c*)(i). Hence, their performance was very poor. Very few of candidates managed to form the required probability and applied the continuity correction in their working.

Answers: (*a*) 157.0; (*b*) 0.6826; (*c*)(i) 68.26; (c)(ii) 0.6549

Question 8

Many candidates chose to answer this question for section B. Good candidates were able to perform well and could easily score full marks in parts (*a*), (*b*), (*c*) and (*d*). They could present their answers systematically. In part (*a*), candidates could give the distribution correctly as $\hat{p} = N\left(0.95, \frac{0.95 \times 0.05}{100}\right)$. However, among the common mistakes done by candidates were; could not relate 95% with proportion of the packets, not using correct notation for sample proportion ($\hat{p}$), instead used p or $\bar{x}$, and used binomial distribution. For part (*b*), almost half of the candidates were able to identify correctly the required probability as $P(\hat{p} \leqslant 90)$ and clearly showed the correct standardisation process before obtaining the answer. However, there were some candidates confusedly started the working wrongly as $P(p_s \leqslant 0.90) \leqslant 200$. Some candidates were able to obtain the correct answer by using calculator without any working. This caused the losing of mark. A handful of the candidates still did the continuity correction before standardising, which was not accepted. In part (*c*), some candidates could state the distribution of the sample mean correctly but they did not state the reason or did not know how to state the reason for the distribution. For part (*d*) of the question, some candidates interpreted the term "...between 201 and 203" as "$P(201 \leqslant \bar{X} \leqslant 203)$" instead of "$P(201 \leqslant \bar{X} \leqslant 203)$" and some candidates used continuity correction before the standardisation. The last part of the question, most candidates knew that the effect of probability would be reduced if the sample size reduced. However, almost all of them were not able to state the correct justification for the effect.

Answers: (*a*) 0.0109; (*d*) 0.9956

Laporan Peperiksaan STPM 2023

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